

P425/2
**APPLIED
MATHEMATICS**

Paper 2
July /Aug. 2025
3 hours



UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer all questions in section A and any five from section B.

All necessary working must be shown clearly.

Silent non – programmable scientific calculators and mathematical tables may be used.

Any extra question(s) attempted in section B will not be marked.

SECTION A (40 MARKS)

1. Given that A and B are events in the same sample space such that $P(A \cap B) = \frac{3}{8}$ and $P(A' \cup B) = \frac{3}{4}$, find;
 - $P(A \cap B')$
 - $P(A)$**(05 marks)**

2. A body falls from rest from the top of a vertical tower, and in its last second it falls $\frac{9}{25}$ of the whole height of the tower. Calculate the height of the tower. **(05 marks)**

3. The marks of students in a Mathematics test are uniformly distributed between 40 and 80. The pass mark is 55.
 - Calculate their mean mark.
 - Three students are picked at random, calculate the probability that all three failed to attain the pass mark.**(02 marks)**
(03 marks)

4. At a car hire service centre the amounts paid to travel to given distances (measured from the centre) are shown in the table above.

Distance (km)	10	15	25
Charge (shs)	30,000	40,000	60,000

Use linear estimation to calculate the;

 - Distance travelled at a cost of 70,000 shillings.
 - Basic fee each passenger pays before leaving the station.**(03 marks)**
(02 marks)

5. The position vector of a particle, t seconds into motion is;

$$\mathbf{r} = 4t^2\mathbf{i} + (6t + 10)\mathbf{j}$$
 metres.
 Calculate the average speed of the particle in the time interval $t = 0$ to $t = 2$ seconds. **(05 marks)**

6. Given that $x = 2.4$, $y = 1.80$ are corrected to the given number of decimal places; compute to maximum error in $x^2 - y$. **(05 marks)**

7. X is a discrete random variable. Study the table below;

x	1	2	3	4	5
$P(X \leq x)$	0.1	0.25	0.55	0.7	1

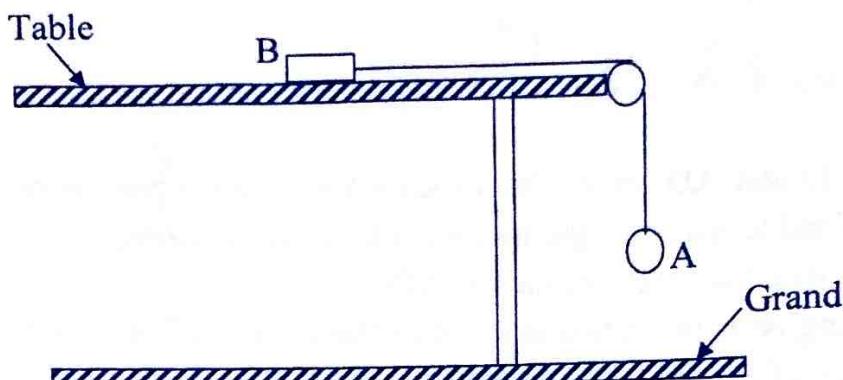
Use the table to;

(04 marks)

- (a) Calculate the mean of X .
(b) Find the 80th percentile.

(01 mark)

8.



A and B are particle of mass 2kg and 3kg respectively.

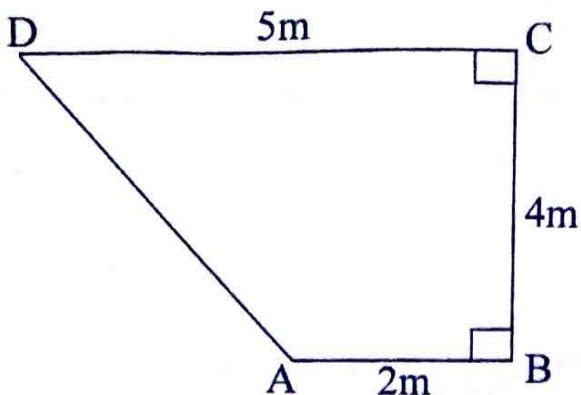
- (a) If particle B is in limiting equilibrium on a horizontal table. Calculate the coefficient of friction at the table and the tension in the string.

(05 marks)

SECTION B

Answer FIVE questions only from this section

9. (a) Derive the iteration formula based on the Newton – Raphson Method for computing the root of the equation $2x = \cos x$. (04 marks)
(b) Construct a flow chart for the process in (a) above.
Using $x_0 = 0.2$; perform a dry run of your flow chart hence find the root correct to 3 dps. (08 marks)
10. In a certain school the ratio of boys to girls is 2:3. Students are picked at random, one at a time with replacement.
(a) Ten students are picked at random. Calculate the probability that at least 6 are boys. (05 marks)
(b) One hundred and fifty students are picked at random; calculate the probability that at least 78 but less than 96 are girls. (07 marks)



AB, BC, CD and AD are uniform rods whose weight per metre is 10N. the rods are joined to make a rigid frame work as shown above.

- (a) Calculate the length of the rod AD. **(02 marks)**
- (b) Taking A as the origin and AB as the x – axis, find coordinates of the centre of gravity of the framework. **(06 marks)**
- (c) The framework is suspended from point B, calculate the angle rod AB will make with the vertical. **(04 marks)**

Number of cars	-10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of days	2	10	6	10	4	0

The table represents the number of cars passing through a check point, recorded over a period of 32 days.

- (a) Construct a histogram for this data, and use it to estimate the modal number of cars passing through the check point.
- (b) Calculate the mean and the variance of the number of cars passing through the point in the given period. Construct a 95% confidence interval for the mean number of cars passing through this point.

13. (a) Calculate the exact value of the integral; **(05 marks)**

$$\int_{-1.5}^0 \frac{1}{\sqrt{4-2x}} dx$$

- (b) Use the trapezium rule with 7 ordinates to

$$\text{evaluate } \int_{-1.5}^0 \frac{1}{\sqrt{4-2x}} dx \text{ to 3 dpls.}$$

Hence compute the absolute error in your answer.

(07 marks)

14. X is a continuous random variable whose C.D.F is given as;

$$F(x) = \begin{cases} 0 & ; x \leq 0 \\ bx^2 & ; 0 \leq x \leq 1 \\ c(10x - x^2 - 5) & ; 1 \leq x \leq 5 \\ 1 & ; x \geq 5 \end{cases}$$

- (a) Find the constants **b** and **c** hence compute the median. **(07 marks)**
(b) Calculate the expectation of X. **(05 marks)**
- (a) The forces $\begin{pmatrix} 6 \\ +5 \end{pmatrix}, \begin{pmatrix} -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix} N$ act at the points (2,2), (-2, 2), (-2, -2) and (2, -2) respectively. Find the equation of the line of action of the resultant force. **(06 marks)**
- (b) To a cyclist travelling southwards at 6ms^{-1} a wind appears to blow in the direction S 030° W; and when he doubles his speed the wind appears to blow in the direction S 060° W. Calculate the actual velocity of the wind. **(06 marks)**
- A particle of mass 2kg initially at rest is acted upon by a force of $48t^2\mathbf{i} + (24 - 8t)\mathbf{j} N$ at any time t seconds. Calculate the;
(a) magnitude of the initial acceleration of the particle
(b) velocity after ts .
(c) work done by the force in the time interval $t = 0$ to $t = 2$ seconds. **(12 marks)**

END

**UTEC 2025 APPLIED MATHEMATICS
PROPOSED GUIDE:**

By TR. E. HENRY

0707443129

1(a) $P(A \cap B) = \frac{3}{8}$, $P(A' \cup B) = \frac{3}{4}$

$P(A \cap B')$:

$$\text{from } P(A' \cup B) = P(A \cap B')' \\ \frac{3}{4} = 1 - P(A \cap B')$$
M₁

$$P(A \cap B') = 1 - \frac{3}{4} \\ = \frac{1}{4}$$
B₁

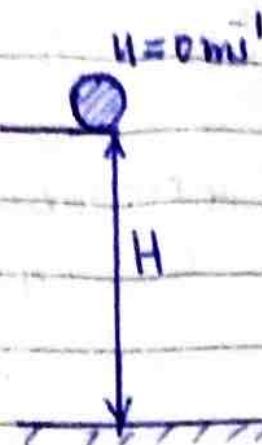
b) $P(A) = P(A \cap B) + P(A \cap B')$
 $= \frac{3}{8} + \frac{1}{4}$

m₁

$$P(A) = \frac{5}{8}$$
A₁

05

(8)



Distance fallen in n seconds; $S_n = \frac{1}{2}gn^2$

$$\Rightarrow \text{Total height, } H = \frac{1}{2}gn^2$$

$$\begin{aligned} \text{But distance fallen in last second} &= S_n - S_{(n-1)} \\ &= \frac{1}{2}gn^2 - \frac{1}{2}g(n-1)^2 \\ &= \frac{1}{2}g(n^2 - (n-1)^2) \\ &= g(n-\frac{1}{2}) \end{aligned}$$
B1

$$\Rightarrow g(n-\frac{1}{2}) = \frac{9}{25} \times \frac{1}{2}gn^2$$
M1

$$9n^2 - 50n + 25 = 0$$
B1

$$n = \frac{50 \pm \sqrt{(-50)^2 - 4(9)(25)}}{2(9)}$$

$$n = 5s \quad n = \frac{5}{9}$$
B1

But $n = 5s$

$$\therefore H = \frac{1}{2} \times 9.8 \times 5^2 \quad \text{or} \quad H = \frac{1}{2} \times 10 \times 5^2$$

$$= 122.5 \text{m} \quad (\text{for } g=9.8 \text{ m/s}^2) \quad = 125 \text{m} \quad (\text{for } g=10 \text{ m/s}^2)$$

3. $[a, b] = [40, 80]$

(i) Mean, $E(x) = \frac{(80+40)}{2}$, from $E(x) = \frac{(b+a)}{2}$ m₁
 $= 60$ B₁

(ii) $P(X < 55) = \int_{40}^{55} \frac{1}{40} dx$ m₁
 $= \frac{1}{40} \left[x \right]_{40}^{55} = \frac{1}{40} (55 - 40)$ B₁
 $= \frac{15}{40}$
 $= \frac{3}{8}$ A₁

4. (g)

D	15	25	d
Shs	40,000	60,000	70,000

$$\frac{25 - 15}{60,000 - 40,000} = \frac{d - 15}{70,000 - 40,000}$$
 B₁

$$\underline{d} = 30 \text{ km}$$
 A₁

cba

(b) Let the charge take a linear form;

$$C = mD + b \text{ (shs)}, \text{ where } C - \text{Cost}$$

Using points (D, C) ;

$$(10, 30,000) \text{ and } (25, 60,000)$$

D - distance

b - Is the basic fee
 m - gradient

$$m = \frac{60,000 - 30,000}{25 - 10}$$

$$m = 2000 \text{ shs/km}$$

m_1

B_1

$$\text{From } C = mD + b, \text{ and point } (10, 30,000)$$

$$b = 30,000 - (2000 \times 10)$$

= 10,000 shillings

B_1

The basic fee the passenger pays is 10,000 shillings

05

$$5 \quad r = 4t^2 \hat{i} + (6t + 10) \hat{j}$$

Average speed = change in displacement
change in time.

$$= \frac{|r(t=2)| - |r(t=0)|}{2 - 0}$$

$$r(t=2) = 16\hat{i} + 22\hat{j}$$

$$r(t=0) = 0\hat{i} + 10\hat{j}$$

$$\text{Average speed} = \frac{\sqrt{16^2 + 22^2}}{2} - \sqrt{10^2} \quad B_1 M_1$$

$$= 8.604 \text{ m s}^{-1}$$

$$x = 2.4, y = 1.80$$

$$e_x = 0.05, e_y = 0.005$$

$$\text{Let } H = x^2 - y$$

$$H_{\max} = x_{\max}^2 - y_{\min}$$

$$= (2.4 + 0.05)^2 - (1.80 - 0.005)$$

$$= 4.2075 \quad B_1 M_1$$

$$H_{\min} = x_{\min}^2 - y_{\max}$$

$$= (2.4 - 0.05)^2 - (1.80 + 0.005)$$

$$= 3.7175 \quad B_1 M_1$$

$$\text{Maximum error} = 4.2075 - 3.7175$$

$$= 0.245 \quad B_1 M_1$$

x	$P(X \leq x)$	$P(X=x)$	$xP(X=x)$
1	0.1	0.1	0.1
2	0.25	0.15	0.3
3	0.55	0.3	0.9
4	0.7	0.15	0.6
5	1	0.3	1.5
			$\sum xP(X=x) = 3.4$

 B_1 B_1

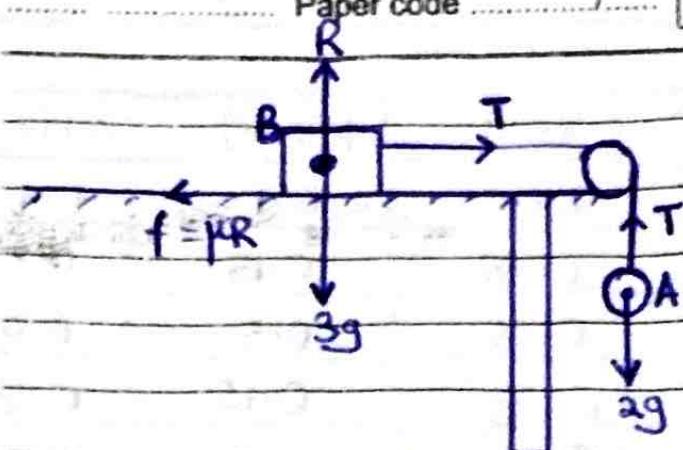
(a) Mean, $\bar{x} = 3.4$ A1

(b) $F(5) = 1 > 0.8$ B1

$\therefore 80^{\text{th}}$ Percentile = 5 A1

 85

8.



Sketch - B1

At limiting equilibrium, $a = 0 \text{ m s}^{-2}$
for the 3kg mass,

$$T = \mu R \quad (1), \quad R = 3g \quad m_1$$

for the 2kg mass;

$$T = 2g$$

$$T = 2 \times 9.8$$

$$T = 19.6 \text{ N} \quad B_1$$

$$\Rightarrow 19.6 = 3 \times 9.8 \mu$$

$$\mu = 0.6667 \text{ or } \frac{2}{3} \quad B_1$$

The Tension in the string is 19.6N and
the Coefficient of friction is $\frac{2}{3}$

1
05

9(a) $f(x) = 2x - \cos x$

$f'(x) = 2 + \sin x$

B₁ - Correct
Diff.

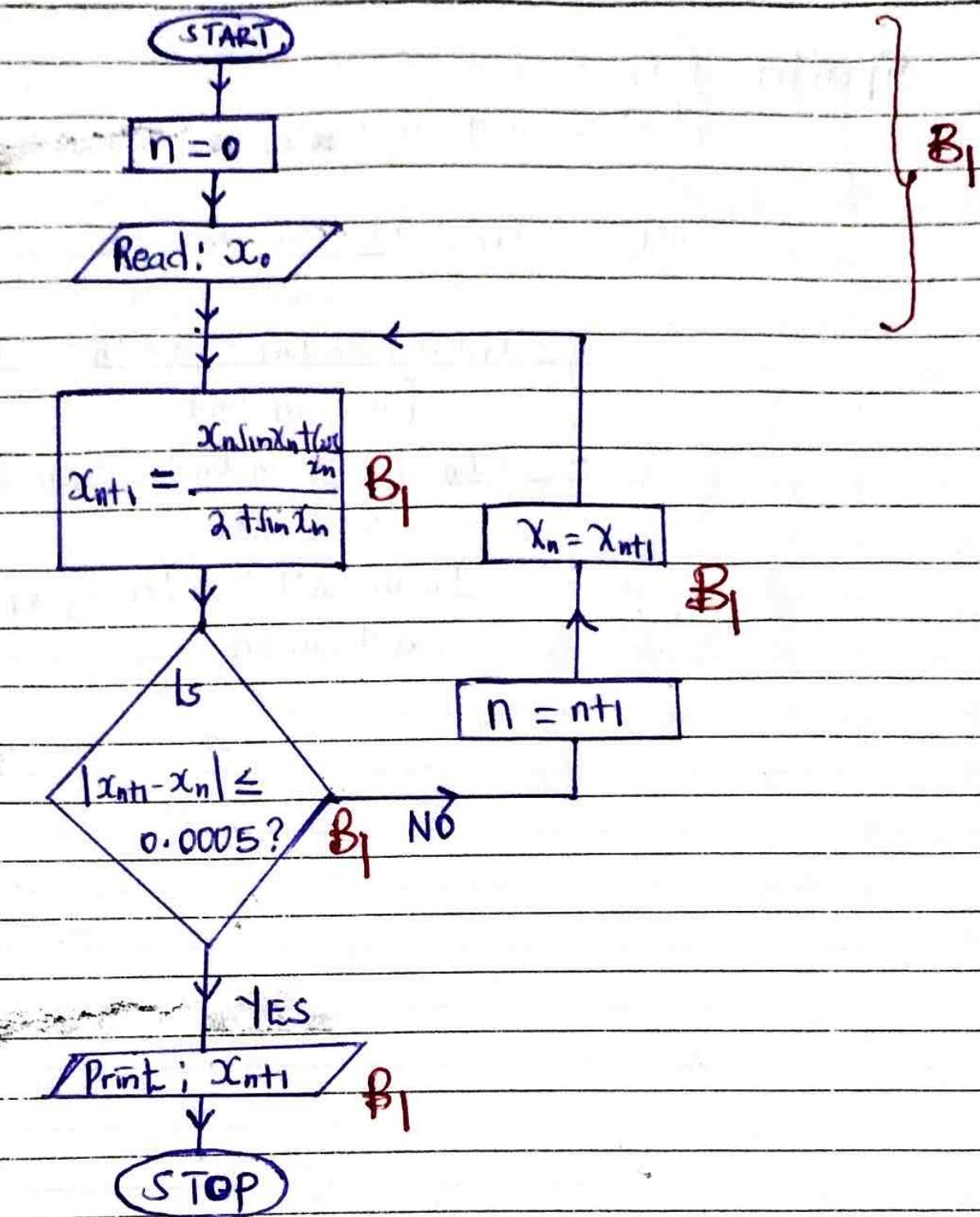
$x_{n+1} = x_n - \frac{(2x_n - \cos x_n)}{2 + \sin x_n}$

$= x_n(2 + \sin x_n) - (2x_n - \cos x_n)$
 $(2 + \sin x_n)$

$= \frac{2x_n + x_n \sin x_n - 2x_n + \cos x_n}{2 + \sin x_n}$

$= \frac{x_n \sin x_n + \cos x_n}{2 + \sin x_n}, n = 0, 1, 2, \dots$ B₁

(b)



Dry Run:

n	x_n	x_{n+1}	$ x_{n+1} - x_n $
0	0.2	0.46383	0.26383
1	0.46383	0.45022	0.01361
2	0.45022	0.45018	0.00004

10 (a) Total ratio = 213

$$= 5$$

P = probability of picking a boy

$$\Rightarrow P = \frac{2}{5}$$

$$q = \frac{3}{5}$$

B1

B1

Let X - represent random variable for number of boy picked:

$$P(X \geq 6) = \sum_{i \geq 6} p_i$$

$$= 0.1662$$

B1 A1A1

Any Alternative approach with correct values:
B1 - for corr. except P

$$(b) n = 150, \mu = np = \frac{3}{5} \times 150$$

$$P = \frac{3}{5}, q = \frac{2}{5}$$

$$\mu = 90$$

$$\sigma = \sqrt{90 \times \frac{2}{5}}$$

$$\sigma = 6$$

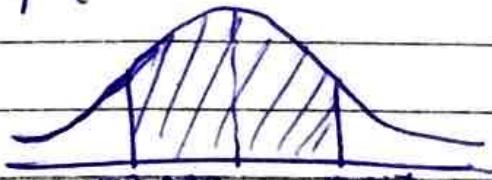
$$P(78 \leq X < 96) = P(77.5 \leq X < 95.5)$$

B1 - for correct Mean

B1 - for correct S.D.

$$= P\left(\frac{77.5 - 90}{6} \leq Z < \frac{95.5 - 90}{6}\right) B1$$

$$= P(-2.083 < Z < 0.917) B1 \rightarrow 3d.p$$



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$$= \Phi(2.083) + \Phi(0.917)$$

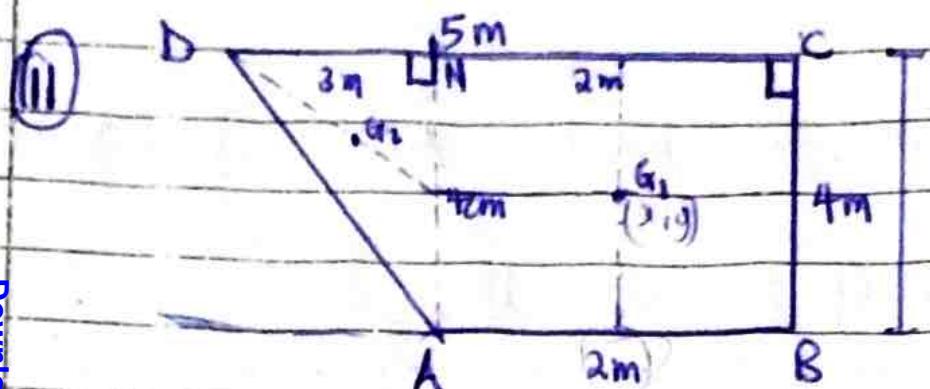
B1 - Correct values

$$= 0.4813 + 0.3203$$

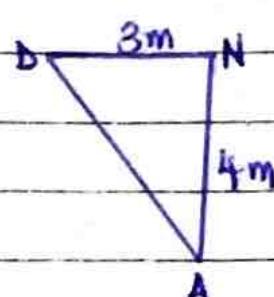
$$= 0.8016$$

A1

12



(a) Length AD:



$$AD^2 = 3^2 + 4^2$$

$$AD = \sqrt{25}$$

$$AD = 5\text{m}$$

Length AD = 5m

m1

t1

52

(b) Let p be the weight per unit area
Taking AB and BC

Figure	Area	Height	x	y
ABCN	$(4 \times 2) = 8$	$8p$	$\frac{1}{2} \times 2 = 1$	$\frac{1}{2} \times 4 = 2$
AND	$(\frac{1}{2} \times 3 \times 4) = 6$	$6p$	$2 + \frac{1}{3}(3) = 3$	$\frac{2}{3} \times 4 = \frac{8}{3}$
Whole	$(8 + 6) = 14$	$14p$	\bar{x}	\bar{y}

B1

Taking moments about AB and BC axes

$$8p \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 6p \begin{pmatrix} 3 \\ 8/3 \end{pmatrix} = 14p \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\begin{pmatrix} 8 \\ 16 \end{pmatrix} + \begin{pmatrix} 18 \\ 16 \end{pmatrix} = \begin{pmatrix} 14\bar{x} \\ 14\bar{y} \end{pmatrix}$$

$$\begin{pmatrix} 26 \\ 32 \end{pmatrix} = \begin{pmatrix} 14\bar{x} \\ 14\bar{y} \end{pmatrix}$$

$$26 = 14\bar{x}$$

$$\bar{x} = \frac{13}{7} \text{ m}$$

$$32 = 14\bar{y}$$

$$\bar{y} = \frac{16}{7} \text{ m}$$

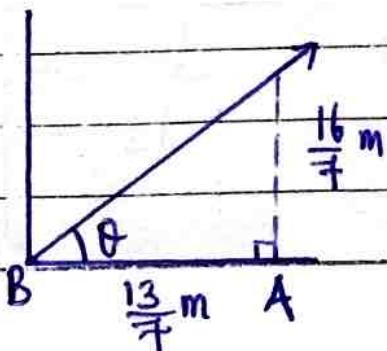
The Co-ordinates of C.O.G are $(\frac{13}{7}, \frac{16}{7})$ m from BC
and $\frac{16}{7}$ m from AB

A1

A1

B6

(c) let θ be angle between AB with the vertical.



$$\tan \theta = \left(\frac{16}{7} \right)$$

$$\theta = 50.91^\circ$$

Q1 - sketch

B1, B1 - Substn
valueA1 - correct
value

04

(12)

Class	f	x	fx	fx^2
0-10	2	5	10	50
10-20	10	15	150	2,250
20-30	6	25	150	3,750
30-40	10	35	350	12,250
40-50	4	45	180	8100
	$\sum f = 32$		$\sum fx = 840$	$\sum fx^2 = 26,400$

$$B_1 - \sum fx^2$$

$$B_1 - \sum fx$$

~~$$n_f = \sum f$$~~

(b) Mean, $\bar{x} = \frac{840}{32}$

$$= 26.25$$

A1

Variance, $\sigma^2 = \left(\frac{26400}{32} \right) - \left(\frac{840}{32} \right)^2$

$$= \frac{2175}{16} \text{ or } 135.9375$$

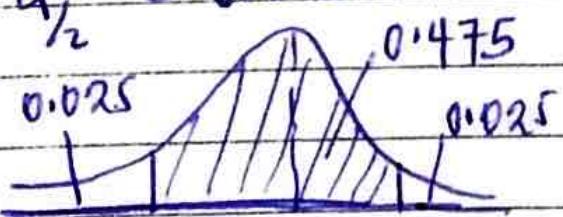
~~mark~~

A1

For 95% Confidence Interval;

$$(1-\alpha) = 0.95$$

$$\alpha_{1/2} = 0.025$$



$$\bar{x}_{\alpha_{1/2}} = \Phi^{-1}(0.475)$$

$$\bar{x}_{\alpha_{1/2}} = 1.96$$

B1

Signature

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$$\mu = \bar{x} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 26.25 \pm 1.96 \cdot \frac{\sqrt{\frac{2175}{16}}}{\sqrt{32}} \quad m_1$$

$$\mu = 26.25 \pm 4.0397$$

$$\text{upper limit} = 26.25 + 4.0397 \\ = 30.2897$$

*Correct
B₁ - for upper
and lower
limit*

$$\text{lower limit} = 26.25 - 4.0397 \\ = 22.2103$$

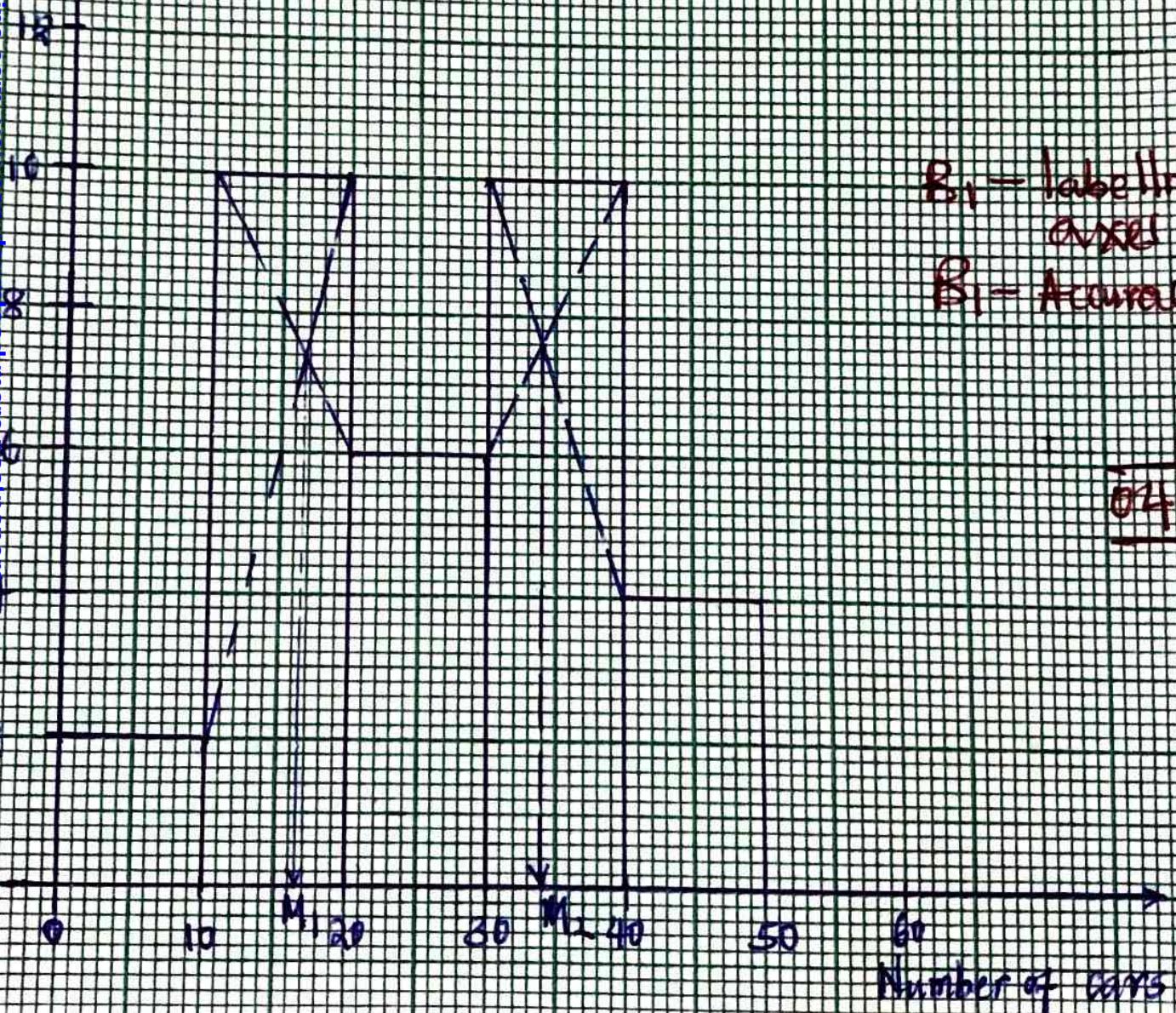
Interval is $[22.2103, 30.2897]$ or A₁

$$22.2103 \leq \mu \leq 30.2897$$

12 (q)

A HISTOGRAM

Frequency



Modal Number of Cars

$$M_1 = 18 \text{ cars } B_1$$

$$M_2 = 34 \text{ cars } B_1$$

B_1 = labelling
axis

B_1 = Accuracy

04

Candidate's Name
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Personal Number	_____	_____	_____	_____

13(a)

$$\int_{-1.5}^0 \frac{1}{\sqrt{4-2x}} dx$$

Let $u = \sqrt{4-2x}$

$$u^2 = 4-2x$$

$$2u du = -2 dx$$

$$dx = -u du$$

Change of limits: m_1

x	-1.5	0
u	$\sqrt{7}$	2

B_1

$$\int_{-1.5}^0 \frac{1}{\sqrt{4-2x}} dx = \int_{\sqrt{7}}^2 \frac{1}{u} \cdot -u du$$

B_1

$$= - \int_{\sqrt{7}}^2 du$$

$$= -u \Big|_{\sqrt{7}}^2$$

$$= -[2 - \sqrt{7}]$$

B_1

$$= 0.64575$$

$$= 0.6458 (4 \text{ dp})$$

A_1

05

Candidate's Name.....

Signature

Random No.

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Subject

Paper code

Personal Number

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(b)

$$h = \frac{1}{4} \quad \text{Let } y = \frac{1}{\sqrt{4-2x}}$$

x	y	
-1.5	0.37796	
-1.25	0.39223	
-1.00	0.40825	
-0.75	0.42640	
-0.5	0.44721	
-0.25	0.47140	
0	0.50000	
Sum	0.87796	2.14549
	B ₁	B ₁

B₁ - Correct
Value of h

$$\int_{-1.5}^0 \frac{1}{\sqrt{4-2x}} dx \approx \frac{1}{8} (0.87796 + 2(2.14549)) m_1 -$$

strictly with
 \approx signs

$$\approx 0.6461175$$

$$\approx 0.646 \quad (3 \text{ d.p.})$$

A₁ - Correct
value to

Absolute error = $|0.6458 - 0.6461| m_1$ 3 d.p.

= 0.0003 A₁

OF

14(a) for $0 \leq x \leq 1$

$$F(1) = b \quad \text{--- (i)} \quad b_1$$

for $1 \leq x \leq 5$

$$F(1) = 4c \quad \text{--- (ii)} \quad B_1.$$

for $x > 5$

$$F(5) = 20c \quad \text{--- (iii)} \quad B_2$$

for $x \geq 5$

$$F(5) = 1 \quad \text{--- (iv)} \quad B_3$$

$$20c = 1$$

$$c = \frac{1}{20} \quad m_1$$

$$\Rightarrow b = 4\left(\frac{1}{20}\right)$$

$$b = \frac{1}{5} \quad m_1$$

$$b = \frac{1}{5}$$

$$c = \frac{1}{20}$$

Median:

$$F(5) = 1 > 0.5$$

$$F(m) = 0.5$$

$$\frac{1}{20}(10m - m^2 - 5) = 0.5 \quad m_1$$

$$m^2 - 10m + 15 = 0$$

$$m = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(15)}}{2(1)}$$

$$m = 8.1623 \quad \text{or} \quad m = 1.8377$$

But $m \neq 8.1623$;

$$\therefore \text{Median} = 1.8377 \quad m_1$$

Signature

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Personal Number	<input type="text"/>	<input type="text"/>	<input type="text"/>

Subject Paper code /

14 (b) ~~for~~ for $0 \leq x \leq 1$

$$f(x) = \frac{d}{dx} \left(\frac{1}{5}x^2 \right)$$

$$= \frac{2}{5}x$$

B1

$$f(x) = \frac{d}{dx} \left(\frac{1}{20}(10x - x^2 - 5) \right)$$

$$= \frac{1}{20}(10 - 2x)$$

$$= \frac{1}{10}(5 - x)$$

$$f(x) = \frac{d}{dx}(1) = 0$$

B1

$$\text{Mean, } E(x) = \int_0^1 \frac{3}{5}x^2 dx + \frac{1}{10} \int_1^5 (5x - x^2) dx$$

$$= \frac{2}{15}x^3 \Big|_0^1 + \frac{1}{10} \left[\frac{5}{2}x^2 - \frac{x^3}{3} \right]_1^5$$

B1

$$= \frac{2}{15}(1 - 0) + \frac{1}{10} \left(\left(\frac{50}{2} - \frac{125}{3} \right) - \left(\frac{5}{2} - \frac{1}{3} \right) \right) m$$

$$= \frac{2}{15} + \frac{28}{15}$$

$$E(x) = 2$$

A1**05**

(15)

$$\text{F} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

M1

$$= \begin{pmatrix} 8 \\ 6 \end{pmatrix} \text{N}$$

B1

$$G = \sum (r \times F)$$

$$= \sum \begin{vmatrix} x & y \\ F_x & F_y \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 2 \\ 6 & 5 \end{vmatrix} + \begin{vmatrix} -2 & 2 \\ -4 & 2 \end{vmatrix} + \begin{vmatrix} -2 & -2 \\ 5 & 0 \end{vmatrix} + \begin{vmatrix} 2 & -2 \\ 1 & -1 \end{vmatrix}$$

M1

$$= (5x2) - (6x2) + (12x2) - (-4x2) + (0 - (-2x5)) + ((-1x2) - (-2x1))$$

$$= -2 + 4 + 10 + 0$$

$$= 12 \text{ Nm}$$

B1

Equation of line of action:

$$G - x\bar{F}_y + y\bar{F}_x = 0$$

$$12 - x(6) + y(8) = 0$$

M1

$$12 - 6x + 8y = 0$$

$$8y = 6x + 12$$

$$4y = 3x + 6$$

A1

06

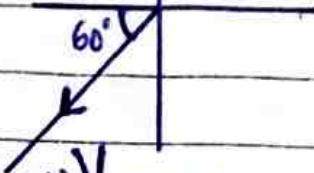
Case One:

(15) (b) Cyclist; Wind.

Let the actual speed
be x ;

$$6 \text{ m/s}$$

$$\underline{V}_c = \begin{pmatrix} 0 \\ -6 \end{pmatrix} \text{ m/s}$$



$$w\underline{V}_c \Rightarrow$$

$$w\underline{V}_c = \begin{pmatrix} -x \cos 60 \\ -x \sin 60 \end{pmatrix}$$

$$w\underline{V}_c = \underline{V}_w - \underline{V}_c$$

$$\begin{aligned} \underline{V}_w &= w\underline{V}_c + \underline{V}_c \\ &= \begin{pmatrix} -x \cos 60 \\ -x \sin 60 \end{pmatrix} + \begin{pmatrix} 0 \\ -6 \end{pmatrix} \end{aligned}$$

$$\underline{V}_w = \begin{pmatrix} -\frac{x}{2} \\ -\frac{\sqrt{3}}{2}x - 6 \end{pmatrix} \quad \text{--- } \textcircled{1}$$

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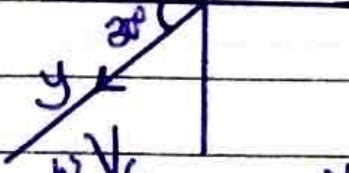
Case two:

Cyclist

Wind;
Let the actual speed be y ;

$$12 \text{ m/s}$$

$$\underline{V}_c = \begin{pmatrix} 0 \\ -12 \end{pmatrix}$$



$$w\underline{V}_c$$

$$w\underline{V}_c = \begin{pmatrix} -y \cos 30 \\ -y \sin 30 \end{pmatrix}$$

$$\begin{aligned} \mathbf{V}_w &= \begin{pmatrix} -y \cos 30^\circ \\ -y \sin 30^\circ \end{pmatrix} + \begin{pmatrix} 0 \\ -12 \end{pmatrix} \\ &= \begin{pmatrix} -\frac{\sqrt{3}}{2}y \\ -\frac{y}{2} - 12 \end{pmatrix} \quad \text{--- (ii)} \end{aligned}$$

B1

$$\text{eqn (i)} = \text{eqn (ii)}$$

$$\begin{pmatrix} -\frac{y}{2} \\ -\frac{\sqrt{3}}{2}y - 6 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2}y \\ -\frac{y}{2} - 12 \end{pmatrix}$$

$$x = \sqrt{3}y \quad \text{--- (iii)}$$

$$-\frac{\sqrt{3}}{2}y - 6 = -\frac{y}{2} - 12 \quad \text{--- (iv)}$$

(iii) into (iv) gives;

$$-\frac{3}{2}y + \frac{y}{2} = -6$$

$$y = 6$$

m₁

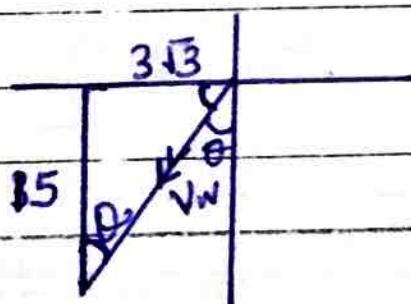
from (i)

$$\mathbf{V}_w = \begin{pmatrix} -\frac{\sqrt{3}}{2}(6) \\ -\frac{6}{2} - 12 \end{pmatrix} = \begin{pmatrix} -3\sqrt{3} \\ -15 \end{pmatrix} \text{ m s}^{-1}$$

B1

$$|\mathbf{V}_w| = \sqrt{(-3\sqrt{3})^2 + (-15)^2}$$

$$= \sqrt{252} \text{ m s}^{-1} \text{ or } 15.8745 \text{ m s}^{-1} \quad \text{B1}$$



$$\tan \theta = \left(\frac{3\sqrt{3}}{15} \right)$$

$$\theta = 53.30^\circ \quad \text{B1}$$

The true velocity of the wind is $\sqrt{252} \text{ m s}^{-1}$ in the direction 553.30°W

(16)

$$\mathbf{F} = 48t^2 \mathbf{i} + (24 - 8t) \mathbf{j} \text{ N}$$

$$m = 2 \text{ kg}$$

$$(a) \quad \mathbf{a} = \frac{\mathbf{F}}{m}$$

$$\mathbf{a} = \frac{1}{2} \begin{pmatrix} 48t^2 \\ 24 - 8t \end{pmatrix} = \begin{pmatrix} 24t^2 \\ 12 - 4t \end{pmatrix} \text{ m s}^{-2} \quad M$$

Initially, at $t=0$

$$\mathbf{a} = \begin{pmatrix} 0 \\ 12 \end{pmatrix} \text{ m s}^{-2} \quad B_1$$

$$|\mathbf{a}| = \sqrt{0^2 + (12)^2} \quad B_1$$

$$= 12 \text{ m s}^{-2} \quad A_1$$

$$(b) \quad \mathbf{v} = \int \mathbf{a} dt$$

$$\mathbf{v} = \int \begin{pmatrix} 24t^2 \\ 12 - 4t \end{pmatrix} dt \quad m_1$$

$$\mathbf{v} = \begin{pmatrix} 8t^3 \\ 12t - 2t^2 \end{pmatrix} + \mathbf{c} \quad B_1$$

$$\text{at } t=0, \mathbf{v} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{v}(t) = \begin{pmatrix} 8t^3 \\ 12t - 2t^2 \end{pmatrix} \text{ m s}^{-1} \quad A_1$$

$$16 \text{ (Q) Work done} = \int_0^2 F \cdot V \, dt$$

$$\begin{aligned} F \cdot V &= (48t^2) \cdot (8t^3) \\ &= (24 - 8t)^5 \cdot (12t - 2t^2) \\ &= 384t^5 + (24 - 8t)(12t - 2t^2) \\ &= 384t^5 + 288t^6 - 144t^4 + 16t^3 \quad B_1 \end{aligned}$$

$$\begin{aligned} \text{Workdone} &= \int_0^2 (384t^5 + 288t^6 - 144t^4 + 16t^3) dt \\ &= [64t^6 + 4t^5 - 48t^3 + 144t^2]_0^2 \quad B_1 \\ &= (4352) - 0 \quad B_1 - \text{Substitution} \\ &= 4352 \quad A_1 \end{aligned}$$

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