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P425/1
PURE
MATHEMATICS
Paper 1
July /Aug. 2025
3 hours

UGANDA TEACHERS' EDUCATION CONSULT (UTEC)

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *questions in section A and any* **five** *from section B.*

All necessary working must be shown clearly.

Silent non – programmable scientific calculators and mathematical tables may be used.

Any extra question(s) attempted in section B will **not** *be marked.*

Turn Ov

$\tan(45+11)$

$\frac{\tan 45 + \tan 11}{1 - \tan 45 \tan 11}$

SECTION A (40 MARKS)

Answer ALL questions in this section

1. Solve the equation $\tan 3\theta = \cot\theta$ for $0° \le \theta \le 180°$. **(05 marks)**

2. Find the equation of the tangent to the curve curve $y = x\ln(x^2 - 3)$ at the point whose x − coordinate is 2. **(05 marks)**

3. Calculate the distance between the parallel lines $3x + 4y + 10 = 0$ and $6x + 8y - 5 = 0$ **(05 marks)**

4. Show that: **(05 marks)**

$$\int_2^3 \frac{(1-x)^2}{1+x^2}\,dx = 1 - \ln 2$$

5. A polynomial $P(x)$ leaves remainders 7 and 1 when divided by $x - 2$ and $x + 1$ respectively. Find the remainder when $P(x)$ is divided by $x^2 - x - 2$. **(05 marks)**

6. Use small changes to evaluate $\sqrt{1.45}$ to 5 dps. **(05 marks)**

7. Solve the inequality $x - 1 \le \frac{2x}{x+2}$ **(05 marks)**

8. Show that the acute angle between two straight lines of gradients m_1 and m_2 is given by $\tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ **(05 marks)**

SECTION B (60 MARKS)

Attempt FIVE question in this section

9. (a) Show that $\sqrt{3}\cos 2\theta + \sin 2\theta = 2\cos(2\theta - 30°)$ hence solve $\sqrt{3}\cos 2\theta + \sin 2\theta = \sqrt{3}$ for $-30° \le \theta \le 270°$. **(06 marks)**

(b) Prove that in any ΔABC; $\frac{a}{\sin A} = 2R$ where R is the radius of the circumscribing circle of the triangle. Hence deduce the value of R, if $b = 6cm$ and $B = 53.13°$. **(06 marks)**

10. (a) Calculate the area bounded by the curve; $y = \sin 2x$ and the axis from $x = 0$ to $x = {}^{x}/_{4}$. **(06 marks)**

(b) By considering a cone of height, h, and base radius r, as a solid of revolution, prove that the volume of the cone, $V = {}^{1}/_{3}\pi r^2 h$.

(06 marks)

11. (a) The sum of n terms of an arithmetic progression is $4n^2 + 6n$. Find the;

(i) First term and the common difference.

(ii) Greatest number of terms whose sum is less than 1,000.

(06 marks)

(b) Use Maclaurin's theorem to find the first three non – vanishing terms of the expansion of $\cos x$ hence evaluate $\cos 9^o$ to 3 decimal places.

(06 marks)

12. (a) Calculate the acute angle between the lines;

$r_1 = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $r_2 = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ **(05 marks)**

(b) Find the Cartesian equation of the plane containing the lines in (a) above.

(07 marks)

13. Given that the points $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ are variable points on the parabola $y^2 = 4ax$.

(a) Show that the equation of the chord PQ is;

$2x - (t_1 + t_2)y + 2at_1t_2 = 0$ **(05 marks)**

(b) If the chord PQ passes throuh the focus $S(a, 0)$ show that $t_1t_2 = -1$ hence;

(i) Find the locus of the mid-point of $\overline{PQ}$

(ii) Show that the tangets at P and Q are perpendicular to each other. **(07 marks)**

14. (a) Evaluate $\int_2^4 \frac{dx}{\sqrt{(4-x^2)}}$ (06 marks)

(b) Given that $y = \sec x + \tan x$ show that;

$\frac{dy}{dx} = y\sec x$ and $\frac{d^2y}{dx^2} = y\frac{dy}{dx}$ (06 marks)

15. (a) $Z = x + iy$ is a variable complex number; such that $|Z - 3 - 4i| = 5$

find the locus of the point (x, y) representing Z. Hence shade the region $|Z - 3 - 4i| > 5$. (06 marks)

(b) Find the cube roots of $-8i$, and represent them on an Argand diagram. (06 marks)

16. (a) The gradient function of a curve at any point (x, y) is $\frac{dy}{dx} = 2x - \frac{16}{x^2}$; given that the $y-$ coordinate of the stationary point is 10, find the equation of the curve. (05 marks)

(b) Lumbers Jacks start to cut down 100 mature trees at a rate which is directly proportional to the square root of the number of trees remaining uncut. After 10 days, 36 trees have been cut down;

(i) Form a differential equation and solve it.

(ii) Calculate the number of days taken to cut down the remaining trees, and state the number of trees being cut down per day by the end of the 25^{th} day. (07 marks)

$y=$

END

300

75