P425/1
PURE MATHEMATICS
MAY 2025

UGANDA ADVANCED CERTIFICATE OF EDUCATION

SET 9

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- Attempt **ALL** the questions in section **A** and only **FIVE (5)** from section **B**.
- All working must be clearly shown.
- Mathematical tables with list of formulae and squared paper are provided.
- Clearly indicate the questions you have attempted in section B in a grid on your answer scripts. **DONOT** hand in question paper.

Qn									
Marks									

SECTION A (40 MARKS)

1. Solve the simultaneous equations:
$3y + x - 3z = -4, \quad 3x - y + 2z = 1, \quad -2x + y + z = 7$ (5 marks)

2. Prove that $\tan^2\left(\frac{\pi}{4} + \theta\right) = \left(\frac{1+\sin 2\theta}{1-\sin 2\theta}\right)$. (5 marks)

3. The first term of an Arithmetic progression and Geometric progression are each $\frac{2}{3}$. Their common difference and common ratio are each to x and the sum of their first three terms are also equal. Find the two possible values of x.
(5 marks)

4. Find the point of intersection between the lines $\mathbf{r} = (-2\mathbf{i} + 5\mathbf{j} - 11\mathbf{k}) + \lambda(3\mathbf{i} + \mathbf{j} + 3\mathbf{k})$, and $\mathbf{r} = (8\mathbf{i} + 9\mathbf{j}) + t(4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$. (5 marks)

5. Evaluate $\int_1^4 \left(\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{2}}\right) dx$, giving your answer in simplified surd. (5 marks)

6. $A(-3, 0)$ and $B(3, 0)$ are fixed points. Show that the locus of a point $P(x, y)$ which moves such that $PB = 2PA$ is a circle and find its centre and radius.
(5 marks)

7. Air is pumped into a spherical balloon at a rate of $256\pi\ cm^3 s^{-1}$. When the radius of the balloon is 15 cm, find the rate at which its radius is increasing. (5 marks)

8. Find the equation of the tangent to the curve $x^2y - xy^2 = 12$ at the point where $(4, 3)$. (5 marks)

SECTION B

9a) Solve the equation: $\sqrt{(y+6)} - \sqrt{(y+3)} = \sqrt{(2y+5)}$. Check your answers.
(6 marks)

b) Solve for x and y: $\begin{matrix} \log(x+y) = 1 \\ 2\log y - \log(30 - x) = 0 \end{matrix}$. (6 marks)

10a) Show that $1 + 2i$ is a root of the equation $2z^3 - z^2 + 4z + 15 = 0$, hence find the other roots. (6 marks)

b) If $z = 1 + 2i$ is a root of the equation $z^3 + az + b = 0$ where a and b are real, find the values of a and b. (6 marks)

11a) Differentiate from first principles $y = \frac{1}{\sqrt{x}}$. (5 marks)

b) A hemispherical bowl is being filled with water at a uniform rate. When the height of the water is $h\ cm$ the volume is $\pi\left(rh^2 - \frac{1}{3}h^3\right)cm^3$, $r\ cm$ being the radius of the hemisphere. Find the rate at which the water level is rising when it is half way to the top, given that $r = 6\ cm$ and the bowl fills in $1\ \text{min}$. (7 marks)

12a) Prove that $\tan\frac{1}{2}(A - B) + \tan\frac{1}{2}(A + \mathbf{B}) = \frac{2\sin A}{\cos A + \cos B}$. (5 marks)

b) If $\tan\alpha = p$, $\tan\beta = q$, $\tan\gamma = r$, prove that $\tan(\alpha + \beta + \gamma) = \frac{p + q + r - pqr}{1 - pr - rq - pq}$, hence,

show that $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{\pi}{4}$. (7 marks)

13a) The profit y generated from the sale is given by the function $y = 72x + 3x^2 - 2x^3$. Calculate how many terms should be sold to receive maximum profit and determine the maximum profit. (5 marks)

b) Find the area enclosed by the curve $y = x(8 - x)$ and the line $y = 12$. (7 marks)

14. Determine the turning points and asymptotes of the curve $y = \frac{4x^2 - 10x + 7}{(x-1)(x-2)}$

hence, sketch the curve. (12 marks)

15a) If the position vectors of points A and B are $2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$ and $-3\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$ respectively, find the position vector of the point P which divides $\mathbf{AB}$ externally in the ratio $5:3$. (6 marks)

b) Find the Cartesian equation of a plane through the origin parallel to the lines

$\mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 4 \\ -5 \\ -8 \end{pmatrix} + \mu\begin{pmatrix} 3 \\ 7 \\ -6 \end{pmatrix}$. (6 marks)

16. Given that the line $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that $c^2 = a^2m^2 + b^2$. Hence, determine the equations of the tangents at the point $(-3, 3)$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. (12 marks)

END

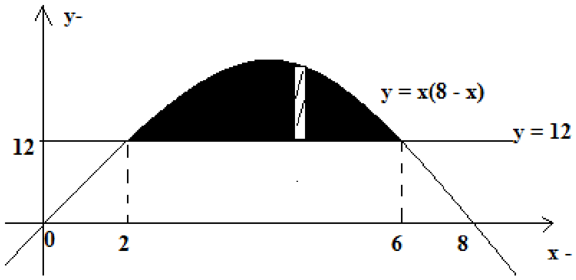
SET 9 MARKING GUIDE 2025

1.	$x + 3y - 3z = -4, (i)\ \ 3x - y + 2z = 1, (ii)\ \ -2x + y + z = 7.\ (iii)$ 3 eqn (i) – eqn (ii), $10y - 11z = -13 \ldots$(iv) 2 eqn (i) + eqn (iii), $7y - 5z = -1 \ldots$(v) 7 eqn (iv) - 10eqn (v), $-27z = -81,\quad z = 3,\quad y = 2,\quad x = -1.$	
2	$\tan^2\left(\frac{\pi}{4} + \theta\right) = \left(\frac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\tan\theta}\right)^2 = \left(\frac{1 + \tan\theta}{1 - \tan\theta}\right)^2 = \left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right)^2$ $= \left(\frac{\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta}{\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta}\right) = \left(\frac{1 + \sin 2\theta}{1 - \sin 2\theta}\right)$	
3	$a + a + x + a\ + 2x\ = a + ax + ax^2$ $3.\tfrac{2}{3} + 3x = \tfrac{2}{3}\left(1 + x + x^2\right)$ $2x^2 - 7x - 4 = 0, \quad \Leftrightarrow \quad (x - 4)(2x + 1) = 0$ $x = 4, \quad x = -\frac{1}{2}$	
4	$\begin{pmatrix}-2\\5\\-11\end{pmatrix} + \begin{pmatrix}3\lambda\\\lambda\\3\lambda\end{pmatrix} = \begin{pmatrix}8\\9\\0\end{pmatrix} + \begin{pmatrix}4t\\2t\\5t\end{pmatrix}$ $\Rightarrow -2 + 3\lambda = 8 + 4t \ldots$(i) $5 + \lambda = 9 + 2t \ \ldots$(ii) $-11 + 3\lambda = 5t \ \ldots$(iii) Eqn(i) – eqn(ii)x3 $\begin{matrix}-2 + 3\lambda & = & 8 + 4t\\ -(15 + 3\lambda) & = & -(27 + 6t)\end{matrix}$ to get $-17 = -19 - 2t$ $\Rightarrow t = -1$ then from eqn(i) $\lambda = 2$ Substitute $t\ \&\ \lambda\ in\ \ (i)$, LHS. $-11 + 6 = -5 =$ RHS $= 4\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}$	

5	$\int_1^4 \left(\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{2}}\right)dx = \int_1^4 \left(x^{-\frac{1}{2}} + \frac{1}{\sqrt{2}}\right)dx$ $= \left[2x^{\frac{1}{2}} + \frac{x}{\sqrt{2}}\right]_1^4$ $= \left(2\left(\sqrt{4} + \frac{4}{\sqrt{2}}\right)\right) - \left(2\sqrt{1} + \frac{1}{\sqrt{2}}\right)$ $= 4 + 2\sqrt{2} - \left(\frac{4+\sqrt{2}}{2}\right)$ $= 2 + \frac{3}{2}\sqrt{2}$	
6	$\sqrt{(x-3)^2 + (y-0)^2} = 2\sqrt{(x+3)^2 + (y-0)^2}$ $x^2 - 6x + 9 + y^2 = 4(x^2 + 6x + 9 + y^2)$ $3x^2 + 3y^2 + 30x + 27 = 0$ $x^2 + y^2 + 10x + 9 = 0$, $(x+5)^2 + (y-0)^2 = 16$ Which is an equation of a circle with centre $(-5,\ 0)$ and $r = 4$	
7	Vol of a sphere is $V = \frac{4}{3}\pi r^3$ and the rate $\frac{dV}{dt} = 256\pi\ cm^3 s^{-1}$ Vol of hemisphere is $\frac{dV}{dr} = 4\pi r^2$, so $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ $\frac{dr}{dt} = \frac{1}{4\pi r^2} \times 256\pi$ for $r = 16$, $\frac{dr}{dt} = \frac{1}{4\pi(15)^2} \times 256\pi = \frac{64}{225}\ cm\ s^{-1}$	
8	$2xy + x^2\frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} = 0$, $\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$ $(4,\ 3)$, $\frac{dy}{dx} = \frac{9-24}{16-24} = \frac{-15}{-8} = \frac{15}{8}$ Equation: $\frac{y-3}{x-4} = \frac{15}{8}$, $8y = 15x - 36$	
9.	$\sqrt{y+6} - \sqrt{y+3} = \sqrt{2y+5}$, square both sides $y + 6 + y + 3 - 2\sqrt{(y+6)(y+3)} = 2y + 5$, $\sqrt{y^2 + 9y + 18} = 2$ $y^2 + 9y + 18 = 4$, $y^2 + 9y + 14 = 0$, $(y+7)(y+2) = 0$, $y = -7$, $y = -2$ Check: $\sqrt{(y+6)} - \sqrt{(y+3)} = \sqrt{(2y+5)}$ $y = -7$, L.H.S $= \sqrt{-1} - \sqrt{-4} \neq \sqrt{-9}$ $\neq$ R.H.S $x = -2$, L.H.S $= \sqrt{4} - \sqrt{1} = \sqrt{1}$ = R.H.S Therefore, $x = -2$, is the only correct solution.	
b)	$x + y = 10$ ....(i) $y = 10 - x$	

	$\frac{y^2}{30-x}=1,\quad y^2=30-x$ ....(ii) $(10-x)^2=30-x$ $x^2-19x+70=0,\quad (x-5)(x-14)=0$ $x=5,\ x=14$ corresponding values are $y=5,\ y=-4$ respectively.	
10 a)	If $z=1+2i$ is a root, then $\bar{z}=1-2i$ the conjugate root is the other root. Sum of roots is $1+2i+1-2i=2$ and product is $(1+2i)(1-2i)=5$ thus the equation is $z^2-2z+5=0$. By long division $z^2-2z+5\overline{)2z^3-z^2+4z+15}$ quotient $2z+3$ $2z^3-4z^2+10z$ $3z^2-6z+15$ $3z^2-6z+15$ Thus $2z+3=0$ gives us $z=-\frac{3}{2}$ Other roots are $2+i,\ -\frac{3}{2}$	
b)	$(1+2i)^3+a(1+2i)+b=0$ $-11-2i+a+2ai+b=0,\quad (-11+a+b)+(2a-2)i=0$ Thus, $2a-2=0,\quad a=1$ $(-11+1+b)=0,\quad b=10$	
11 a)	$y+\partial y=\frac{1}{\sqrt{x+\partial x}},\quad \partial y=\frac{1}{(\sqrt{x+\partial x})}-\frac{1}{\sqrt{x}}$ $\partial y=\frac{\sqrt{x}-\sqrt{x+\partial x}}{\sqrt{x}(\sqrt{x+\partial x})},\quad \partial y=\frac{\sqrt{x}-\sqrt{x+\partial x}}{\sqrt{x}(\sqrt{x+\partial x})}\times\frac{(\sqrt{x}+\sqrt{x+\partial x})}{(\sqrt{x}+\sqrt{x+\partial x})}$ $\partial y=\frac{x-x-\partial x}{\sqrt{x}(\sqrt{x+\partial x})(\sqrt{x}+\sqrt{x+\partial x})},\quad \frac{\partial y}{\partial x}=\frac{-1}{\sqrt{x}(\sqrt{x+\partial x})(\sqrt{x}+\sqrt{x+\partial x})}$ As $\partial x\to 0,\ \frac{\partial y}{\partial x}\to\frac{dy}{dx}$, so $\frac{dy}{dx}=\frac{-1}{2x^{3/2}}$	
b)	Vol of hemisphere is $V=\frac{2}{3}\pi r^3$ and the rate $\frac{dV}{dt}=\frac{\pi r^3}{90}cm^3s^{-1}$ $V=\pi\left(rh^2-\frac{1}{3}h^3\right),\quad \frac{dV}{dh}=\pi\left(2rh-h^2\right)$ So, $\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt},\quad \frac{dh}{dt}=\frac{1}{\pi(2rh-h^2)}\times\frac{\pi r^3}{90}$ for $r=6,\ h=3$ $\frac{dh}{dt}=\frac{216}{90\times 27}=\frac{4}{45}cm\ s^{-1}$	

12 a)	From the L.H.S $\tan\frac{1}{2}(A-B)+\tan\frac{1}{2}(A+\mathbf{B})=\dfrac{\sin\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A-B)}+\dfrac{\sin\frac{1}{2}(A+B)}{\cos\frac{1}{2}(A+B)}$ $=\dfrac{\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)+\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)}$ $=\dfrac{\frac{1}{2}(\sin A-\sin B)+\frac{1}{2}(\sin A+\sin B)}{\frac{1}{2}(\cos A+\cos B)}$ $=\dfrac{2\sin A}{\cos A+\cos B}$ as the R.H.S	
b)	From the L.H.S, $\tan(\alpha+\beta+\gamma)=\tan((\alpha+\beta)+\gamma)$ $=\dfrac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma}$ $=\dfrac{\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-\left[\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\right]\tan\gamma}$ $=\dfrac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\gamma-\tan\alpha\tan\beta-\tan\beta\tan\gamma}$ $\tan(\alpha+\beta+\gamma)=\dfrac{p+q+r-pqr}{1-pr-rq-pq}$ $\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{4}+\tan^{-1}\dfrac{2}{9}=\dfrac{\pi}{4}$ $\alpha+\beta+\gamma=\tan^{-1}\left(\dfrac{p+q+r-pqr}{1-pr-rq-pq}\right)$, $=\tan^{-1}\left(\dfrac{\frac{1}{3}+\frac{1}{4}+\frac{2}{9}-\frac{1}{3}\times\frac{1}{4}\times\frac{2}{9}}{1-\frac{1}{3}\times\frac{1}{4}-\frac{1}{3}\times\frac{2}{9}-\frac{2}{9}\times\frac{1}{4}}\right)=\tan^{-1}\dfrac{\frac{85}{108}}{\frac{85}{108}}$ $=\tan^{-1}1=\dfrac{\pi}{4}$	
13 a)	$y=72x+3x^2-2x^3$ $\dfrac{dy}{dx}=72+6x-6x^2$, for max $\dfrac{dy}{dx}=0$ So, $72+6x-6x^2=0$, thus $x^2-x-12=0$ $(x-4)(x+3)=0$ so $x=4,\ x\neq-3$ For $x=4$, $y=288+48-128=208$	
b)	For points of integration to get the limits of integration, $12=x(8-x)$, $x^2-8x+12=0$ $(x-6)(x-2)=0$ so $x=6,\ x=2$, points are $(2,\ 12)$, & $(6,\ 12)$	



$$A=\int_2^6 \left(8x-x^2-12\right)dx$$

$$=\left[4x^2-\frac{x^3}{3}-12x\right]_2^6$$

$$=\left(144-72-72\right)-\left(16-\tfrac{8}{3}-24\right)=\frac{32}{3}\,sq.units$$

14 Turning points, $\dfrac{dy}{dx}=\dfrac{\left(x^2-3x+2\right)\left(8x-10\right)-\left(4x^2-10x+7\right)\left(2x-3\right)}{\left(x^2-3x+2\right)^2}$

For $\dfrac{dy}{dx}=0$, we have,

$8x^3-10x^2-24x^2+30x+16x-20-8x^3+12x^2+20x^2-30x-14x+21=0$

To get, $-2x^2+2x+1=0$

$$x=\frac{-2\pm\sqrt{2^2-\left(-4\times2\times1\right)}}{-4}=\frac{1}{2}\pm\frac{\sqrt{3}}{2}\quad \text{so } x=-0.366,\ x=1.366$$

Thus, $(1.366,\ -3.464)\max$, $(-0.366,\ 3.464)\min$

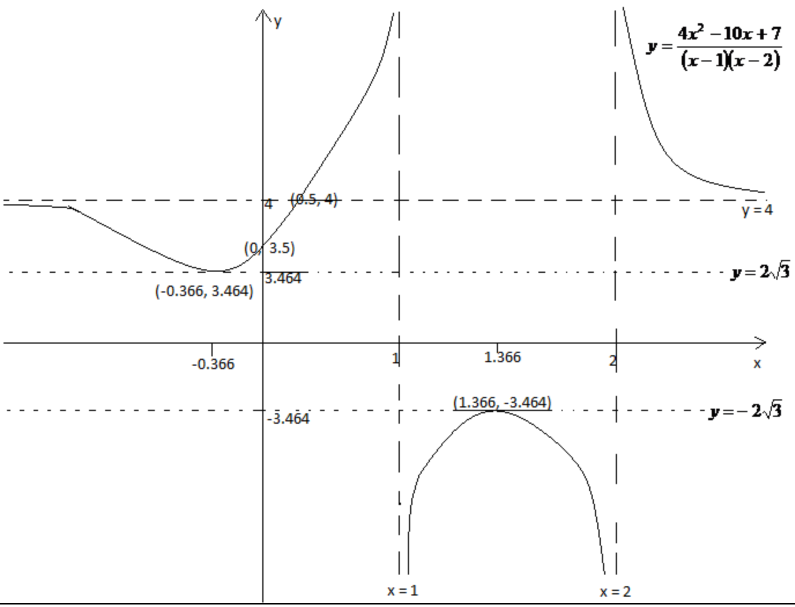
Intercepts: $x=0,\ y=\dfrac{7}{2}$ so $(0,\ 3.5)$

$y=0,\ 4x^2-10x+7=0$ has no real roots since $(-10)^2-(16\times7)<0$

Vertical asymptotes: $x=1,\ x=2$

Horizontal asymptote, $y=4$, $4x^2-12x+8=4x^2-10x+7$,

$x=\dfrac{1}{2}$, thus curve crosses horizontal asymptote at $(0.5,\ 4)$



15 a)	$\mathbf{AP}:\mathbf{PB} = -5:3 \; or \; 5:-3$ $\Rightarrow +3\,\mathbf{AP} = -5\mathbf{PB}, \quad 3(\mathbf{AO}+\mathbf{OP}) = -5(\mathbf{PO}+\mathbf{OB})$ $3\mathbf{OP} - 5\mathbf{OP} = -5\mathbf{OB} + 3\mathbf{OA}$ $-2\mathbf{OP} = -5\begin{pmatrix}-3\\2\\8\end{pmatrix} + 3\begin{pmatrix}2\\4\\6\end{pmatrix}, \quad \mathbf{OP} = -\frac{1}{2}\left[\begin{pmatrix}15\\-10\\-40\end{pmatrix} + \begin{pmatrix}6\\12\\18\end{pmatrix}\right]$ $\mathbf{OP} = \begin{pmatrix}-21/2\\-1\\11\end{pmatrix}$ so $\mathbf{OP} = \frac{-21}{2}\mathbf{i} - \mathbf{j} + 11\mathbf{k}$.	
b)	Let $\mathbf{u} = \begin{pmatrix}1\\-1\\-2\end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix}3\\7\\-6\end{pmatrix}$ and let $\mathbf{a} = \begin{pmatrix}x\\y\\z\end{pmatrix}$ be the normal to $\mathbf{u}$ & $\mathbf{v}$. $\Rightarrow x - y - 2z = 0 \;\ldots(\text{i})$ and $3x + 7y - 6z = 0 \;\ldots(\text{ii})$ Using (i) and (ii) $7x - 7y - 14z = 0$, $3x + 7y - 6z = 0$, to get $x = 2z$, thus from (i) $y = 0$ So $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = 2z\mathbf{i} + 0\mathbf{j} + z\mathbf{k} = z(2\mathbf{i} + \mathbf{k})$ $\Rightarrow$ normal vector is $2\mathbf{i} + \mathbf{k}$ But $ax + by + cz = ax_o + by_o + cz_o$ $\Rightarrow \quad 2x + 0 + z = 0 + 0 + 0$ $\therefore \; 2x + z = 0$.	
16	$\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$ $(a^2m^2 + b^2)x^2 + 2a^2cmx + a^2c^2 - a^2b^2 = 0$ For a tangent, $B^2 - 4AC = 0$, so $4a^4c^2m^2 - 4a^2(a^2m^2 + b^2)(c^2 - b^2) = 0$ for $4a^2b^2c^2 \neq 0$, then $a^2c^2m^2 - a^2c^2m^2 + a^2m^2b^2 - b^2c^2 + b^4 = 0$ $c^2 = a^2m^2 + b^2$ For $\frac{x^2}{16} + \frac{y^2}{9} = 1, \quad a^2 = 16, \quad b^2 = 9$ $c^2 = 16m^2 + 9$, and for $(-3,\ 3)$, $-3 = 3m + c$, so, $c = -3 - 3m$ $16m^2 + 9 = 9 + \mathbf{18m} + \mathbf{9m^2}$, $\quad m(7m - 18) = 0$ thus $m = 0, \quad \& \; m = \frac{18}{7}$, similarly, $c = -3, \quad \& \; c = -\frac{75}{7}$ Equations are $y = -3$ and $7y = 18x - 75$	