P425/1
Pure math
Paper 1
3 hours
ARPIL 2025

SET 7

UGANDA ADVANCED CERTIFICATE OF EDUCATION

INSTRUCTIONS TO CANDIDATES:

- Attempt **all** the eight questions in section A and any **five** questions from section B.
- Clearly show all the necessary working
- Begin each answer on a fresh sheet of paper
- Silent, simple non-programmable scientific calculators may be used.

SECTION A (40 MARKS)

1. Solve the simultaneous equations: $2x - y + 2z = 6$ and $\frac{x+2}{3} = \frac{y+2}{4} = \frac{z+2}{5}$.

2. Solve the equation: $(1 - \sin x)^2 + (1 + \cos x)^2 = 1$ for $0^o \le x \le 180^o$.

3. The points A, B, C have position vectors $4\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}$, $6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ and $\mathbf{i} + 10\mathbf{j} + 3\mathbf{k}$ respectively. Show that angle ABC is a right angle.

4. Form a differential equation given that, $y = 4\sin(2t + \alpha)$ and state the order.

5. The distance of the point $P(2, \ -1)$ from the line $y = \frac{3}{4}x + c$ is twice its distance from the line $y = -\frac{3}{4}x$, find the value of c.

6. Using the substitution of $u = \sin\theta$, evaluate $\int_0^{\pi} \sin^2\theta \cos^3\theta \, d\theta$

7. Solve the equations: $\log_b a + 2\log_a b = 3$ and $\log_9 a + \log_9 b = 3$, given that $a \ne b$.

8. The radius of a sphere increases at a rate of $0.01\, cm\ s^{-1}$. Find the rate at which:

i) surface area increases   ii) volume increases when the radius is $21\, cm$

SECTION B (60MARKS)

9. Express $\frac{2x^2-5x+7}{(4x^2-9)(x+2)}$ in partial fractions, hence, expand $\frac{2x^2-5x+7}{(4x^2-9)(x+2)}$ in ascending powers of x up to the term in x^2.

10. Given the curve $y=\frac{(x-2)^2}{x+1}$ determine the turning points of the curve, the asymptotes and sketch the curve.

11a) Given that the point C divides the line $\overline{AB}$ in the ratio $1:2$ and the position vectors of A and C are $-\mathbf{4i}-\mathbf{3j}+\mathbf{5k}$ and $3\mathbf{i}-2\mathbf{j}+12\mathbf{k}$ respectively, find the coordinates of point B.

b) A plane contains the points $A(4,\ -6,\ 5)$ and $B(2,\ 0,\ 1)$. A perpendicular to the plane from the point $P(0,\ 4,\ -7)$ intersects the plane at point C. Find the Cartesian equation of the line $\overline{PC}$.

12a) Solve the equation: $7\cot x-\cos ecx=5$, for $0\le x\le\frac{3}{2}\pi$.

b) Given triangle PQR, prove that $\tan\left(\frac{Q-R}{2}\right)=\frac{q-r}{q+r}\cot\frac{P}{2}$, hence, solve the triangle with two sides $5\,cm$ and $7\,cm$ and the included angle is 45^o.

13a) Use De Moivre's theorem to express $\tan 4\theta$ in terms of $\tan\theta$.

b) Solve: $z^3+8=0$.

14a) Given that $y = 1 - \cos\theta$ and $x = \sin\theta$, show that $\left(\frac{d^2y}{dx^2}\right)^2 = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^2$.

b) The displacement x of a particle at any time t is given by $x = \sin t$. Find the mean value of its velocity over the interval $0 \le t \le \frac{\pi}{2}$ with respect to:

i) time

ii) displacement

15a) If the line $2x - 3y + 26 = 0$ is a tangent to the circle $x^2 + y^2 - 4x + 6y - 104 = 0$, find the coordinates of the point of contact.

b) Find the equation of the circle which passes through the points $(1,\ 1)$ and $(1,\ -1)$ and is orthogonal to $x^2 + y^2 = 4$.

16. In a certain city, the rate at which buildings are collapsing is proportional to those that have collapsed. If initially B_o is the number that have already collapsed.

a) Show that $B = B_o e^{kt}$, where k is a constant and B_o is the number of buildings that have already collapsed.

b) If the number of collapsed buildings doubled the initial number in 10 years, find the value of k.

c) Determine the number of buildings that would have collapsed after 30 years in terms of the initial number B_o.

END