P425/1
Pure Mathematics
Paper 1
ARPIL 2025
3 hours

SET 6
Uganda Advanced Certificate of Education
Pure Mathematics Paper 1
Time: 3 Hours

INSTRUCTIONS TO CANDIDATES:

- *Answer **all** the **eight** questions in Section **A** and only **five** questions in Section **B**.*
- *Indicate the five questions attempted in section B in the table aside.*
- *Additional question(s) answered will **not** be marked.*
- ***All** working **must** be shown clearly.*
- *Graph paper is provided.*
- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A (40 MARKS)

Qn 1: An arithmetic progression contains $\boldsymbol{n}$ terms. The first term is 2 and its common difference is $\frac{2}{3}$. If the sum of the last four terms is 72 more than the sum of the first four terms, find $\boldsymbol{n}$. [5marks]

Qn 2: Find the equation of a circle which touches the line $3x + 4y = 9$ has a centre $(4, -7)$. [5marks]

Qn 3: Differentiate $\mathbf{cos}\,\boldsymbol{x}$ from first principles. [5marks]

Qn 4: Four letters of the word "**HYPERBOLA**" are to be arranged in a row. In how many of these arrangements are the vowels separate? [5marks]

Qn 5: Solve for x, $2\sin^2\left(\frac{x}{2}\right) - \cos x + 1 = 0$, where $0 \leq x \leq 2\pi$. [5marks]

Qn 6: Prove that the integral of $\text{cosec}\left(\frac{x}{2}\right)$ for x between π and $\frac{4\pi}{3}$ is $\ln 3$. [5marks]

Qn 7: Find the shortest distance of a point $A(1, 6, 3)$ from the line $\underset{\sim}{\boldsymbol{r}} = \underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}} + \underset{\sim}{\boldsymbol{k}} + \beta\left(-\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}} + 2\underset{\sim}{\boldsymbol{k}}\right)$. [5marks]

Qn 8: The surface area of a sphere is decreasing at a rate of $0.9\ \text{m}^2/\text{s}$ when the radius is 0.6 m. Find the rate of change of the volume of the sphere at this instant. [5marks]

SECTION B (60 MARKS)

Question 9:

(a). If the roots of the equation $x^2 + (x + 1)^2 = k$ are α and β;

(i). Prove that $\alpha^3 + \beta^3 = \frac{1}{2}(1 - 3k)$.

(ii). Find a quadratic equation whose roots are α^3 and β^3.

(b). (i). Given that $|x| < \frac{1}{2}$, expand $\frac{1+5x}{\sqrt{1+2x}}$ upto the term in x^3.

(ii). By substituting $x = 0.04$ in (b)(i) above, deduce the approximation of $\frac{1}{\sqrt{3}}$ correct to 4 decimal places. [12marks]

Question 10:

Given that $y = \frac{\sin x - 2\sin 2x + \sin 3x}{\sin x + 2\sin 2x + \sin 3x}$

(i). Prove that $y + \tan^2\left(\frac{x}{2}\right) = 0$, and hence express the exact value of $\tan^2 15°$ in the form $p + q\sqrt{r}$ where p, q and r are integers.

(ii). Hence find the value of x between $0°$ and $360°$ for which $2y + \sec^2\left(\frac{x}{2}\right) = 0$. [12marks]

Question 11:

Given the curve $f(x) = \frac{2x^3 - x^2 - 25x - 12}{x^3 - x^2 - 5x + 5}$;

(a). Find the:

(i). value of x for which $f(x) = 0$.

(ii). assymptotes for $f(x)$.

(iii). x and $f(x)$ intercepts for the curve.

(b). Sketch the curve. [12marks]

Question 12:

A point representing the complex number Z moves such that $\left|\frac{Z-2}{Z-4}\right| > \frac{1}{2}$

(i). Prove that the locus of Z is a circle.

(ii). Find the centre and radius of this circle.

(iii). Represent Z on the argand diagram.

(iv). State the least and greatest values of $|Z|$. [12marks]

Question 13:

(a). Given two vectors $\underset{\sim}{\boldsymbol{a}} = 3\underset{\sim}{\boldsymbol{i}} - 12\underset{\sim}{\boldsymbol{j}} + 4\underset{\sim}{\boldsymbol{k}}$ and $\underset{\sim}{\boldsymbol{b}} = \underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{k}}$; find:

(i). the angle between $\underset{\sim}{\boldsymbol{a}}$ and $\underset{\sim}{\boldsymbol{b}}$,

(ii). a vector that makes a right angle with $\underset{\sim}{\boldsymbol{a}}$ and with $\underset{\sim}{\boldsymbol{b}}$.

(b). Find the equation of the plane passing through the points $\boldsymbol{A}(1, 1, 0)$, $\boldsymbol{B}(3, -1, 1)$, $\boldsymbol{C}(-1, 0, 3)$ and find the shrotest distance of the point $(3, 2, 1)$ to the plane. [12marks]

Question 14:

(a). Using calculus of small increments, or otherwise, find $\sqrt{98}$ correct to one decimal place. [4marks]

(b). Use Maclaurin's theorem to expand $\ln(1 + ax)$, where a is a constant. Hence or otherwise expand $\ln\left(\frac{1+x}{\sqrt{1-2x}}\right)$ up to the term in x^3. For what value of x is the expansion valid? [8marks]

Question 15:

A tangent to the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ at a point, $P(6\cos\theta\,, 4\sin\theta)$ meets the minor axis at $\boldsymbol{A}$. If the normal at $\boldsymbol{P}$ meets the major axis at $\boldsymbol{B}$, find the:

(i). Coordinates of $\boldsymbol{A}$,

(ii). Coordinates of $\boldsymbol{B}$,

(iii). Locus of the midpoint of $\boldsymbol{AB}$. [12marks]

Question 16:

(a). Find the general solution of

$$(x^2 + 1)\frac{dy}{dx} + 2x - 2xy = 0$$

(b). A moth ball evaporates at a rate proportional to its volume, losing half of its volume every 4 weeks. If the volume of the moth ball is initially $15\ \text{cm}^3$ and becomes ineffective when its volume reaches $1\ \text{cm}^3$, how long is the moth ball effective? [12marks]

END