P425/1
PURE MATHEMATICS
PAPER 1
MARCH 2025
3 HOURS



Uganda Advanced Certificate of Education
SET 4
PURE MATHEMATICS
Paper 1
3 hours

**INSTRUCTIONS TO CANDIDATES:**

- Attempt **ALL** the **EIGHT** questions in section **A** and any **FIVE** from section **B**.
- All working must be clearly shown.
- Begin each question on a fresh sheet of paper.
- Mathematical tables with list of formulae and squared paper are provided.
- Silent, non-programmable calculators should be used.

## Section A (40 marks)

1. Find the values of $\lambda$ for which $10x^2 + 4x + 1 = 2\lambda x(2 - x)$ has equal roots.

2. Prove that the maximum volume of a right circular cylinder which can be inscribed in a sphere of radius $r$ is $\frac{4\pi r^3}{3\sqrt{3}}$ .

3. A polynomial $P(x)$ when divided by $(x - 1)$ leaves a remainder 3 and when divided by $(x - 2)$, leaves a remainder 1. Find the remainder when $P(x)$ is divided by $x^2 - 3x + 2$.

4. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{dx}{4-5\sin^2 x}$ correct to 4 decimal places.

5. Given that $x = t^2 - t$ and $y = t^2 - 4$. Find $\frac{d^2y}{dx^2}$ in terms of $t$.

6. Find the angle between the line $\boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k} + \lambda(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$ and the plane $2x - y + z = 4$.

7. Solve the equation: $\cos 2\theta + \cos 3\theta + \cos\theta = 0$ for $0 \leq \theta \leq 2\pi$.

8. Find the equation of the normal to the ellipse at the point $(5\cos\theta, 3\sin\theta)$.

## Section B (60 marks)

9. (a). Prove that the equation of the tangent to the parabola at the point $P(at^2, 2at)$ is $ty = x + at^2$.
   (b). The tangents to the above parabola at the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ intersect at R. Find the coordinates of R.
   (c). If the tnagents at P and Q are inclined to one another at the angle of $45°$, show that the locus of R is $y^2 = x^2 + 6ax + a^2$.

10. (a). Find the perpendicular distance of the point $P(3, 0, 1)$ rom the line $\boldsymbol{r} = \boldsymbol{i} - 2\boldsymbol{j} + \lambda(3\boldsymbol{i} + 4\boldsymbol{j} + 12\boldsymbol{k})$.

(b). (i). A point $A(1, 2, -3)$ and two non-parallel vectors $2\boldsymbol{i} + 4\boldsymbol{k}$ and $-\boldsymbol{i} - 4\boldsymbol{j} + 2\boldsymbol{k}$ lie on a plane. Find the Cartesian equation of the plane.

(ii). Find the angle between the line in (a) above and the plane.

11. (a). Given that $z = \frac{(1-i)(\sqrt{3}-i)}{(1-i\sqrt{3})}$ , express $z$ is polar form.

(b). Show that the locus of $\left|\frac{z-1}{z+1}\right| = 2$ is a circle. State its centre and radius.

(c). Solve the equation $z^2 - 4(1+i)z + 9 + 8i = 0$.

12. Given the curve $y = \frac{x^2-5x+6}{x-1}$ ;

(i). State all the asymptotes of the curve.

(ii). Find the turning points of the curve.

(iii). State the intercepts of the curve.

(iv). Hence sketch the curve.

13. (a). Prove that $\frac{1-\cos 2A+\sin 2A}{1+\cos 2A+\sin 2A} = \tan A$.

(b). Solve the equation: $\sin 2\theta + 1 = \cos 2\theta$ for $0° \leq \theta \leq 360°$.

14. (a). If $e^x \frac{d^2y}{dx^2} + 2e^x \frac{dy}{dx} + 2e^x y = 0$.

(b). Integrate:

(i). $\int x^2 e^{-3x}\ dx$

(ii). $\int \sin 3x \cos x\ dx$

15. Express $\frac{x^4+2x}{(x-1)(x^2+1)}$ as partial fractions. Hence evaluate $\int \frac{x^4+2x}{(x-1)(x^2+1)}\ dx$ to 4 significant figures.

16. (a). Solve the differential equation $(1+\cos 2\theta)\frac{dy}{dx} = 2$ for $y\left(\frac{\pi}{4}\right) = 1$.

(b). The rate at which a boy loses temperature at any instant is proportional to the amount by which the temperature of the body, at that instant exceeds that of its surroundings. A container of hot liquid is placed in a room of temperature $18°\mathrm{C}$ and in 6 minutes, the liquid cools from $82°\mathrm{C}$ to $50°\mathrm{C}$. How long does it take for the liquid to cool to $20°\mathrm{C}$ ?

**END**