S6 PRE-MOCKS SET 3 2024

PURE MATHEMATICS P425/1

Instructions

- Attempt all questions in section A any five questions in section B.
- Show your workings clearly.

SECTION A(40 MARKS)

1. The sum on n terms of a particular series is given by $S_n = 17n - 3n^2$.
   (i)Find the nth term of the series. (3 mks)
   (ii) Show that the series is an $A.P$ (2 mks)
2. Solve the equation : $\cos(4\theta + \frac{\pi}{2}) = \cos(2\theta - \frac{\pi}{6})$ for $0 \leq \theta \leq 2\pi$. (5 mks)
3. Find the vector equation of a line which passes through the point $A(4, -1)$ and parallel to the vector $\boldsymbol{i} - \boldsymbol{j}$. Hence show that the point $\boldsymbol{i} + \boldsymbol{2j}$ lies on the line. (5 mks)
4. For what values of μ does the equation $x^2 - 2x + 1 = \mu(x - 3)$ has equal roots. (5mks)
5. The volume, V of a sphere increases by 6% when the radius is increased by $k\%$. Find the value of k. (5 mks)
6. Show that the point $P(2, -4)$ lies on the parabola $y^2 = 8x$ and hence find the equation of the normal to the parabola at the point A. (5 mks)
7. Given that $x = \sqrt{\left(\frac{y^2-2}{5}\right)}$, show that $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 5$. (5 mks)
8. Solve the differential equation; $\frac{dy}{dx} = e^x(1 + y^2)$ for $y(ln2) = 1$. (5 mks)

SECTION B(60 MARKS)

9. (a) If $\log_2 p = x$ and $\log_4 q = y$ .Find the values of x and y given that $p^2q = 16$ and $\frac{p}{q^2} = \frac{1}{32}$. (6 mks)
   (b) Solve the simultaneous equations; $9^x = 27^y$,$64^{xy} = 512^{x+1}$ (6 mks)
10. (a) Find the gradient of the curve represented by the parametric equations $x = \frac{t^2}{1+t}$ and $y = \frac{1-t}{1+t}$.Hence find the equation of the tangent at the point where $t = 2$. (5 mks)
    (b) Sketch the curve $y = 4x(x - 4)$ and hence find the area bounded by the curve and the $x - axis$. (7 mks)

11. (a) Solve the equation; $\cos^{-1} x + \cos^{-1} x\sqrt{8} = \frac{\pi}{2}$. (5 mks)

(b) Given that $\sin\alpha + \sin\beta = p$ and $\cos\alpha + \cos\beta = q$, show that $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{p}{q}$. Hence show that $\cos(\alpha+\beta) = \frac{q^2-p^2}{q^2+p^2}$. (7mks)

12. (a) Find the term independent of x in the expansion of $\left(3x - \frac{2}{x^2}\right)^{18}$. (5 mks)

(b) In the expansion of $(1+ax)^n$,the first three terms are $1 - \frac{5x}{2} + \frac{75x^2}{8} + \cdots$, find the values of n and a. Hence state the range of values of x for which the expansion is valid. (7 mks)

13. (a) Solve $\int_{ln2}^{ln3} \frac{1}{e^{-x}+1} dx$ in the form $ln\left(\frac{A}{B}\right)$ where A and B are constants (6 mks)

(b) Find the indefinite integral; $\int \frac{1}{x^2-4x+29} dx$. (6 mks)

14. (a) Show that the curve $4x^2 + 9y^2 = 36$ is an ellipse. Hence find it's eccentricity, centre and foci. (5 mks)

(b) Find the coordinates of the point at which the normal to the rectangular hyperbola $xy = 8$ at the point $(4,2)$ cuts the tangent to the hyperbola $16x^2 - y^2 = 64$ at the point $(2\frac{1}{2}, 6)$. (7 mks)

15. (a) Given the vertices of a triangle ABC are $A(1,-2,3)$, $B(-3,-2,1)$ and C is the origin, show that it's area is $\sqrt{29}$ square units. (6 mks)

(b) Given that the two lines $\boldsymbol{r_1} = (-\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}) + \alpha(\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k})$ and $\boldsymbol{r_2} = (-3\boldsymbol{i} + b\boldsymbol{j} + 7\boldsymbol{k}) + \beta(\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k})$ intersect at a point P.Find;

(i) the values of α, β and b.

(ii) the coordinates of P (6 mks)

16. The rate at which the number of residents in a certain village in Amuru increases is directly proportional to the number present at that time. If the number increases from 1000 to 2000 residents in one year.

(a) Form a differential equation for the information above. (6 mks)

(b) How many residents will be in the village after one and half years. (3 mks)

(c) How many years will it take for the residents to become 8000 . (3 mks)

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SET BY **@OJP**