P425/1
PURE
MATHEMATICS
Paper 1
June, 2024
3hrs

SET 1

SMACON EXAMINATIONS 2024
Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

<u>INSTRUCTIONS TO CANDIDATES</u>:

- *Attempt **all** the **eight** questions in **Section A** and **Not** more than **five** from*
- ***Section B.***
- *Any additional question(s) will not be marked.*
- *All working must be shown clearly.*
- *Silent non-programmabe calculators and mathematical tables with a list of formulae may be used.*
- *Graph papers are provided.*

SECTION A: (40MARKS)

*Answer **all** the questions in this Section.*

1. Find the sum of the numbers between 5 and 250 which are exactly divisible by 4. *(5marks)*

2. Given that the line; $\frac{x-3}{4} = \frac{y-4}{-3} = \frac{z+3}{4}$ meets the plane $4x - 3y - 4z = 3$ at M. Find the coordinates of M. *(5marks)*

3. Use the substitution $x = sin\theta$ to find the integral; $\int \frac{2x^3}{\sqrt{1-x^2}}\, dx$. *(5marks)*

4. Express $tan(45^0 + x)$ in terms of $tanx$. Hence prove that; $tan75^0 = 2 + \sqrt{3}$. *(5marks)*

5. Given $A(3,4)$ and $B(-2,3)$, find the equation of the locus of points $P(x,y)$ which divide AB in the ratio $2:1$. *(5marks)*

6. A women football team manager intends to take 18 players for a tournament. The manager has 2 goal keepers, 8 defenders, 4 mid fielders and 8 strikers. In how many ways can the team be chosen if it must contain both goal keepers, atleast 3 midfielders and 7 strikers. *(5marks)*

7. Solve the differential equation; $Cosecx\frac{dy}{dx} = e^x cosecx + 3x$. *(5marks)*

8. Solve for x in the equation; $log_{(x+3)}(2x + 3) + log_{(x+3)}(x + 5) = 2$. *(5marks)*

SECTION B (60MARKS)

*Attempt any **five** questions from this Section.*

9. Given that $f(x) = \frac{x^3+2x^2+61}{(x+3)^2(x^2+4)}$, express $f(x)$ in partial fraction. Hence evaluate; $\int_0^1 f(x)dx$. *(12marks)*

10. $P(ap^2,\ 2ap)$ and $Q(aq^2,\ 2aq)$ are two variable points on the parabola $y^2 = 4ax$. If PQ subtends a right angle at the origin, prove that $pq = -4$.

    a) Prove that PQ passes through a fixed point on the axis of the parabola.

    b) The tangents at P and Q meet at R, find the equation of the locus of R. *(6marks)*

11. a) Differentiate $tan^{-1}\left(\frac{\sqrt{InX}}{e^{2x}}\right)$. *(6marks)*

b) Evaluate the integral; $\int_0^{\frac{\pi}{6}} \frac{2cos\theta + sin\theta}{cos\theta - sin\theta}\, d\theta$. *(6marks)*

12. a) P is the foot of the perpendicular from the point $A(1, 1, 1)$ to the line $\frac{X-1}{2} = \frac{y-1}{1} = \frac{Z-2}{1}$. Determine the perpendicular distance of A from the line to 4 $dp's$. *(5marks)*

b) Given the points $A(-1, 2, 3)$ and $P(2, 3, 4)$. If the point $B(a, 2a, 3)$ lies on the plane $2x - 3y + 4z + 8 = 0$. Find the value of a and the angle between AP and AB. *(7marks)*

13. a) Solve the equation $tan\theta - cot\theta = -1$ for $0^0 \leq \theta \leq 360^0$. *(5marks)*

b) Prove that $\frac{Sin3\theta}{1+2cos2\theta} = Sin\theta$. Hence show that $Sin15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}}$. (7marks)

14. a) Prove that $log_a^b = \frac{1}{log_b a}$. hence solve the equation $log_2 x + log_x 2 = 2.5$. *(5marks)*

b) A polynomial is given by $P(x) = x^3 + Ax^2 + x - 6$. The ratio of the remainder when $P(x)$ is divided by $(X + 1)$ to the remainder when divided by $(x - 2)$ is $-1{:}5$. find the value of A. *(7marks)*

15. a) If $Z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$, express Z in modulus argument form. *(5marks)*

b) Use demoiver's theorem to prove that $2cos\theta = Z + \frac{1}{Z}$ then $2cosn\theta = Z^n + \frac{1}{Z^n}$. Hence solve the equation $5Z^4 - 11Z^3 + 6Z^2 - 11Z + 5 = 0$. *(7marks)*

16. a) Determine the nature of the turning points of the curve $y = x(1-x)^2$. *(5marks)*

b) The acceleration of a particle is proportional to 2t-3. If the velocity increases from 4ms^{-1} to 8ms^{-1} in the first 2 seconds of motion, find;

i) its initial acceleration *(5marks)*

ii) the velocity after 5 seconds. *(2marks)*

**** ***END*** ****