OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL PURE MATHEMATICS SEMINAR QUESTIONS 2023

ALGEBRA

1. (a) Find the square root of $14 + 6\sqrt{5}$

(b) Given $y = x + \frac{1}{x}$, Solve for x in the equation $4x^4 + 17x^3 + 8x^2 + 17x + 4 = 0$

(c) Find the first three terms of the Binomial expansion of $\sqrt{(1+x)(1+x^2)}$

(d) Given that the equation $x^2 + 3x + 2 = 0$ has roots k and l, find the equation whose roots are $\frac{k}{l^2}$ and $\frac{l}{k^2}$

(OLAN)

2. (a) Prove by induction that for all positive integer$\sum_{r=1}^{n}(3r+1)(r+2) = n(n+2)(n+3)$

(b) Prove by induction that for all positive odd integers, n, $f(n) = 4^n + 5^n + 6^n$ is divisible by 15

(c) Use Demoivre's theorem to prove that $sin5\theta = 5sin\theta - 20sin^3\theta + 16sin^5\theta$

(d) Hence or otherwise, find the distinct roots of the equation $2 + 10x - 40x^2 + 32x^5 = 0$ giving your answer correct to 3 dps where appropriate. **(JINJA COLLEGE)**

3. (a) The sum of n terms of the sequence is $S_n = 2^{2n} - n$ where n a natural numbers is 61. Find the first three terms of the sequence.

(b) From a class of 14 boys and 10 girls, 10 students are to be selected for a competition in which 5 boys and 5girls or 2girls and 8boys are to go for it. In how many ways can they be selected?

(c) The roots p and q of a quadratic equation are such that $p^3 + q^3 = 4$ and $pq = \frac{1}{2}(p^3 + q^3) + 1$. Find a quadratic equation with integral coefficients whose roots are p^6 and q^6.

(d). Mary operates an account with a bank which offers a compound interest of 5% per annum. She opened the account at beginning of 2019 with shs 800,000 and continue to deposit the same amount at beginning of every year. How much will she receive at the end of 2022. If she made no withdrawal with in this period?

(ST CHARLES LWANGA BUKERERE)

4. (a) Solve the inequality $\frac{x+3}{x-2} \geq \frac{x+1}{x-2}$

(b) Given the curve $y = \frac{x^2+x-2}{x^3-7x^2+14x-8}$

(i) Give the coordinates of the hole

(ii) Find the equations of the asymptotes

(iii) Determine the turning points and their nature

(iv) Find the intercepts and sketch the curve.

(ST MICHEAL S.S SONDE)

TRIGONOMETRY

5. (a) Given the function, $f(x) = \frac{3}{13+6sinx-5cosx}$ use the substitution $t = tan\left(\frac{x}{2}\right)$, to show that $f(x)$ can be written in the form $\frac{3(1+t^2)}{2\,(3t+1)^2+6}$

(b) Given that $cot^2\theta + 3cosec^2\theta = 7$, show that $tan\theta = \pm 1$

(c) (i) Express the function $y = 3cosx - \sqrt{3}sinx$ in the form $Rcos(\theta + \alpha)$where R is a constant and $0 \leq \alpha \leq 2\pi$

(ii) Hence find the coordinates of the minimum point of y

(iii) State the values of x at which the curve cuts the $x - axis$

(CODE HIGH SCHOOL)

6. (a) Show that $cos3\theta = 4cos^3\theta - 3cos\theta$. Hence if $cos\theta = \frac{1}{2}(a + \frac{1}{a})$, prove that $cos3\theta = \left(a^3 + \frac{1}{a^3}\right)$

(b) Given that A, B, and C are angles of a triangle, prove that; $Sin^2 {}^{A}/_{2} + Sin^2 {}^{B}/_{2} + Sin^2 {}^{C}/_{2} = 1 - 2Sin {}^{A}/_{2}\, Sin {}^{B}/_{2}\, Sin {}^{C}/_{2}$

(c) Given that $Sin(\theta + \alpha) = a$ and $Sin(\theta + \beta) = b$. Show that $Cos2(\alpha - \beta) - 4abCos(\alpha - \beta) = 1 - 2a^2 - 2b^2$.

(JINJA PROGRESSIVE)

7. (a) Given that $cos45^0 = \frac{1}{\sqrt{2}}$. Show without using a calculator or tables that $Sin\left(292\frac{1}{2}^0\right) = -\frac{1}{2}\sqrt{2+\sqrt{2}}$

(b) If $cos\alpha - cos\beta = \frac{2}{3}$ and $sin\alpha - sin\beta = \frac{5}{6}$, find the value of;

(i) $Sin\frac{1}{2}(\alpha + \beta)$ (ii) $Cos(\alpha + \beta)$

(c) Given that A,B and C are angles of $\frac{a^2+b^2-c^2}{a^2-b^2+c^2} = tanBcotC$.

(d) Find all the sides of a triangle ABC whose area is $1008cm^2$ and $a = 65cm,\ b + c = 97cm$.

(MENTHA HIGH SCHOOL)

8. (a) Given that X, Y, Z are angles of a triangle, Prove that $tan\left(\frac{X-Y}{2}\right) = \left(\frac{x-y}{x+y}\right)Cot\left(\frac{Z}{2}\right)$, hence solve the triangle if $x = 9cm, y = 5.7cm\ and\ Z = 57^0$.

(b) Prove that $\frac{Cos11^0+sin11^0}{Cos11^0-Sin11^0} = tan56^0$

(c) In triangle ABC, $AB = x - y, BC = x + y\ and\ CA = x$ Show that $CosA = \frac{x-4y}{2(x-y)}$

(d)(i) Show that $tan^{-1}\left(\frac{1}{2}\right) + tan^{-1}\left(\frac{1}{5}\right) = tan^{-1}\left(\frac{7}{9}\right)$.

(ii) Prove that $Sin(2sin^{-1}x + cos^{-1}x) = \sqrt{1-x^2}$

(GAYAZA ROAD TRIANGLE KIWENDA)

ANALYSIS

9. (a) Use the method of small changes to find the value of $\frac{1}{\sqrt{0.97}}$ correct to 3dps

(b) Evaluate $\int_0^1 \frac{8x-8}{(x+1)^2(x-3)^2}\,dx$

(c)(i) Given that $f(x) = \frac{x^4+x^3-6x^2-13x-6}{x^3-7x-6}$, Express $f(x)$ into partial fractions

(ii) Hence evaluate $\int_4^5 f(x)\,dx$

(d) Evaluate $\int_0^{\frac{\pi}{4}} \frac{4}{1+cos2x}\,dx$

(MUKONO KINGS)

10.(a)(i). On the same axes, sketch the curve $y = x(x+2)$ and $y = x(4-x)$.

(ii). Find the area enclosed by the two curves in $a(i)$ above

(iii). Determine the volume generated when the area enclosed by the two curves in $a(i)$ above is rotated about the $x-axis$.

(b) Evaluate $\int_2^6 \frac{\sqrt{x-2}}{x}\,dx$

(c) A match box consists of an outer cover open at both both ends into which a rectangular box without a top. The length of the box is 1.5 times the width. The thickness of the material is negligible and the volume of the match box is $25cm^3$. If the width is xcm, find interms of x the area of the material used. Hence show that if the least area of the material is to be used to make the box then the length should be approximately $3.7cm$

(OLAM)

11. (a) In order to post a parcel, the sum of the circumference of a cylindrical parcel and its height should add up to $6cm$. Find the dimensions of the largest parcel that can be accepted.

(b) A sample of bacteria in a sealed container is being studied. The number of bacteria, P in thousands, is given by the differential equation $(1+t)\frac{dp}{dt} + p = (1+t)\sqrt{t}$ where t the time in hours after the start of the study is. Initially, there are exactly 5,000 bacteria in the container.

(i) Determine, according to the differential equation, the number of bacteria in the container $8hours$ after the start of the study.

(ii) Find, according to the differential equation, the rate of change of the number of bacteria in the container 4 $hours$ after the start of the study.

(NAMRUTH HIGH SCHOOL)

12. (a) Given that $y = \log_e \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$, prove that $\frac{dy}{dx} = -secx$

(b) Solve the differential equation $x\frac{dy}{dx} = 2x - y$

(c) In an agricultural plantation the proportion of the total area that has been destroyed by a bacterial disease is x. The rate of the destruction of the plantation is proportional to the product of the proportion already destroyed and that not yet. It was initially noticed that half of the plantation had been destroyed by the disease and that at this rate another quarter of the plantation would be destroyed in the next 6*hours*.

(i) Form a differential equation relating x and time t

(ii) Calculate the percentage of the population destroyed 12 *hours* after the disease was noticed.

(ST JAMES BUDDO)

VECTORS

13. (a) Find the angle $\alpha = \langle BAC$ of the triangle ABC whose vertices are $A(1,\ 0,\ 1)$, $B(2, -1,\ 1)$ and $C(-2,\ 1,\ 0)$.

(b) The planes $P_1 and\ P_2$ are respectively given by the equations $r = 2i + 4j - k + \lambda(i + 2j - 3k) + \mu(-i + 2j + k)$ and $r.(2i - j + 3k) = 5$ where λ and μ are scalar parameters. Find;

(i) The Cartesian equation for plane, P_1 .

(ii) To the nearest degree, the acute angle between P_1 and P_2

(iii) The coordinates of the point of intersection of the plane, P_1 and the line $\frac{x-1}{5} = \frac{y-3}{-3} = \frac{z+3}{4}$

(WAKISO HILLS)

14(a) Given the line $r = (3 + 2\mu)i + (1 - \mu)j + (-2 + 2\mu)k$. Find the;

(i) Value of d if the line is in the plane $r.(i - 2j - 2k) = d$.

(ii) Distance of the point (3,1,7) from the line.

(b) Given the points $A(2, -5,3)\ and\ B(7,0, -2)$, find the coordinates of point C which divides AB externally in the ratio 3: 8.

(c) Show that the points $P(1,2,3), R(3,8,1)\ and\ T(7,20, -3)$ are collinear.

(PRIDE COLLEGE)

15(a) Find the equation of the plane which contains a point $A(2,1, -2)$and is parallel to the plane $x - y - 4z = 3$

(b) If a line $\frac{x}{2} = \frac{y+3}{0} = \frac{z-1}{1}$ intersects with the plane in (a) above, find the;

(i) Point of intersection.

(ii) The angle between the line and the plane.

(WHITE ANGELS HIGH SCHOOL)

16 (a). Find the vector equation of the line of intersection between the planes $r.\begin{pmatrix}1\\1\\-3\end{pmatrix} = 6$ and $r.\begin{pmatrix}2\\-1\\1\end{pmatrix} = 4$.

(b) Using the dot product only, find the equation of the plane containing points $A(0,1,1), B(2,1,0)$ and $C(-2,0,3)$.

(c) A straight line joining the points $(2,1,4)$ and $(a-1,4,-1)$ is parallel to the line joining points $(0,2,\ b-1)$ and $(5,3,\ -2)$. Find the values of a and b.

(TALENTS COLLEGE)

COORDINATE GEOMETRY

17.(a) The points A, B, and C have coordinates $A(-3,\ 2),\quad B(-1,-2)$ and $C(0,\ n)$, where n is a constant. Given that $\overline{BC} = \frac{1}{5}\overline{AC}$, Find possible values of n.

(b) A straight line L, passes through the point (-2, 1) and makes an angle of 45^0 with the horizontal.

(i) Find the equation of line L.

(ii) Given that the line L, intersects the $x-axis$ at A and the $y-axis$ at B. Find the distance AB.

(c) Find the coordinates of the circumcenter of the triangle ABC with vertices $A(3,2), B(1,4)$ and $C(5,4)$.

(d) Given that $A(0,-5), B(-7,2)$ and $C(2,11)$ are vertices of a parallelogram $ABCD$, find the coordinates of the point D.

(MAKERERE HIGH SCHOOL)

18.(a) Find the equation of the circle whose centre is at (5,4) and touches the line joining (0,5) and (4,1)

(b) A circle that passes through the points $A(3,4)$ and $B(6,1)$ and the equation of the tangent to this circle at A is the line $2y = x + 5$. Find;

(i) The coordinates of the centre of the circle.

(ii) The radius of the circle.

(iii) The equation of the circle.

(c) If $y = mx$ is a tangent to a circle $x^2 + y^2 + 2fy + c = 0$ prove that $c = \frac{f^2m^2}{1+m^2}$. Hence find the equation of the tangents from origin to the circle $x^2 + y^2 - 10y + 20 = 0$.

(d) Show that the circles whose equations are $x^2 + y^2 - 4y - 5 = 0$ and $x^2 + y^2 - 8x + 2y + 1 = 0$ cut orthogonally.

(QUEENS S.S)

19.(a) P and Q are two points whose coordinates are $(at^2, 2at), (\frac{a}{t^2}, \frac{-2a}{t})$ respectively and S is a point$(a, 0)$. Show that $\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}$

(b) Prove that $x = 3t^2 + 1$ and $2y = 3t + 1$ are parametric equations of a parabola. Find its Vertex, Focus and length of latus rectum.

(c) The point $P(at^2, 2at)$ is on the parabola $y^2 = 4ax$. The chord OQ passes through the origin$O$. The tangent at P is parallel to the chord OQ. The tangents to the parabola at P and Q meet at a point R. Determine the coordinates of points Q and R in terms of a and t.

(ST JOHNS NTEBETEBE)

20.(a) Show that $x^2 + 2y^2 + 6x - 8y = 7$ is an ellipse and hence determine its centre and eccentricity

(b) Points S and S' are the foci of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. Find the coordinates of S and S'

(c) The conic section below has eccentricity $e < 1$ and equation $\frac{x^2}{9} + y^2 = 1$. Find the value of e

(d).If the line $y = mx + c$ is a tangent to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $c^2 = b^2 + a^2m^2$.

Hence determine;

(i) Equations of the four common tangents to the ellipses $\frac{x^2}{23} + \frac{y^2}{3} = 1$ and $\frac{x^2}{14} + \frac{y^2}{4} = 1$

(ii) The equations of the tangents at the point $(-3,3)$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(PAUL MUKASA HIGH SCHOOL)

21(a). Find the equation of the tangent to the hyperbola $x^2 - 9y^2 = 1$ at $P(sec\beta,\ \frac{1}{3}tan\beta)$

(b) The tangent at any point $P\left(ct, \frac{c}{t}\right)$ on the hyperbola $xy = c^2$ meets x and y at A and B respectively. O is the origin.

(i) Prove that $AP = PB$

(ii) Prove that the area of triangle AOB is constant

(iii) If the hyperbola is rotated through an angle of -45^0 about O, find the new equation of the curve.

(GOOD CHOICE HIGH SCHOOL)

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