NABISUNSA GIRL'S SCHOOL.

A – LEVEL MATHEMATICS SEMINAR 2025.

P425/1 PURE MATHEMATICS.

PAPER STRUCTURE

	SECTION A	SECTION B
TRIGONOMETRY	1	1
VECTORS	1	1
GEOMETRY	1	1
ALGEBRA	2	2
ANALYSIS (CALCULUS)	3	3

GEOMETRY	ALGEBRA	ANALYSIS
Coordinate geometry Circles and locus Parabolas Ellipse Hyperbola	Permutations and combination QEs and polynomials. logarithms and indices. Partial fractions complex numbers. Inequalities equations Ratio theorem. binomial and pascals triangle. Series	Integration Differentiation Curve sketching, I and II Differential equations Inequalities Rates of change Small changes Maclaurin's theorem

ALGEBRA.

1. If $x^2 + y^2 = 7xy$, prove that $\log(x+y) = \log 3 + \frac{1}{2}\log x + \frac{1}{2}\log y$.
2. The quadratic polynomial $P(x)$ leaves a remainder of -6 on division by $(x+1)$, a remainder of -5 on division by $(x+2)$ and no remainder on division by $(x+3)$. Find $P(x)$ and solve the equation $P(x) = 0$.
3. Solve the equation. $7^{2x-3} - 7(7^{x-2}) + 1 = 0$.
4. The sum of the first terms of an AP and GP is 12. The sum of the second terms of the terms of the same AP and GP is 27. The sum of the third terms of the same AP and GP is 66. If they have the same first term, find the sum of the fourth term.
5. Given that $Z_1 = 3 + i$ and $Z_2 = x + i$ and $arg(Z_1Z_2) = \frac{\pi}{4}$, Find the value of x.
6. Given that $Z = 1 + i$. is a root of the equation $Z^4 - 4Z^3 + 3Z^2 + 2Z - 6 = 0$ and hence find other roots.
7. Solve the inequality: $\frac{x+1}{2x-1} \le \frac{1}{x-3}$.
8. Solve the equation: $\sqrt{x+6} + \sqrt{4-x} = \sqrt{1-3x}$.
9. The second, $4th$ & $8th$ terms of an AP are in a GP. If the sum of the 3rd & 5th term is 20, find the sum of the first four terms.
10. Find the coefficient of $\frac{1}{x^5}$ in $\left(2x + \frac{1}{x}\right)^7$.
11. Find the locus of Z if the $arg\left(\frac{Z-3}{Z-2i}\right) = \frac{\pi}{4}$.
12. A committee of 4men and 3 women is to be formed from 10men and 8men. In how many ways can the committee be formed.
13. Expand $(1+x)^{-1}$ and $(1-2x)^{\frac{1}{2}}$ including the 4th term and hence show that $(1+x)^{-1} - (1-2x)^{\frac{1}{2}} \approx \frac{3x^2}{2}$.
14. The sum of the first n –terms of a certain progression is $n^2 + 5n$ for all integral value of n. find the first three terms and prove that the progression is an AP.
15. Solve the equation. $\frac{16^x - 4^x}{4^x + 2^x} = 5(2^x) - 8$.

16.Find the values of x and y if they are real in $\frac{2y+4i}{2x+y} - \frac{y}{x-i} = 0$

17.The gradient of the curve $y = ax^2 + bx + c$ is $4x + 2$. The function has a minimum value of 1. find the value of a and b, c.

18.Expand $\sqrt{\frac{1+[illegible]x}{1-[illegible]x}}$ as far as the term including x^3. Taking the first three terms, evaluate $\sqrt{14}$ to 3sf.

19.Prove by induction

$$1.2 + 2.3 + 3.4 + \cdots + n(n+1) = \frac{1}{3}n(n+1)(n+2).$$

20.Given the complex number Z, $Z = \frac{(3i+1)(i-2)^4}{i-3}$, determine the $argZ$ and $|Z|$.

21.Prove that if $\frac{Z-6i}{Z+8}$ is purely real, the locus of the point representing Z is a circle.

22.Find the remainder when the polynomial $f(x)$ is divided by $x^2 - 9$ given that $f(x)$ divided by $x - 3$ has a remainder 2 and the remainder -3 when divided by $x + 3$.

23.Find the number of ways for which letters of the word SWALLOW can be arranged if

(i) the Ls are always together.

(ii) the two W's must not be together

24.If Z_1 and Z_2 are complex number, solve the simultaneous equations.

$4Z_1 + 3Z_2 = 23$ and $Z_1 + iZ_2 = 6 + i8$ in $x + iy$.

25. Express the following into partial fractions

(i) $\frac{x-[illegible]}{3x^2-11x+10}$.

(ii) $\frac{x+4}{(x+1)(x-2)^2}$.

(iii) $\frac{6x^2-3x+1}{(4x+1)(x^2+1)}$.

(iv) $\frac{(x-2)^2}{x^3+1}$.

(v) $\frac{x^4-x^3+x^2+1}{x^3+[illegible]}$.

26.The polynomial $p(x) = 3x^3 + ax^2 - 7x + b$ has a factor of $x + 1$. When divided by $x - 2$, the remainder is 30. Determine the values of a and b. Hence factorize the polynomial completely.

27.Use Maclaurin's theorem to expand $ln(2 + 3x)$ as far as the term x^4. Hence evaluate $ln(2.03)$ correct to 4dp.

28.Find the square root of $(12i - 5)$.

29.Show the region represented by $|Z - 2 + i| < 1$ on the argand diagram.

30.Expand $(8 + 3x)^{\frac{1}{3}}$ in ascending powers of x as far as the term in x^3, stating the values of x for which the expansion is valid. Hence obtain the approximate value $\sqrt[3]{8.72}$ correct 4dp.

31.Find the square root of;

32.$6 + 14\sqrt{5}$.

33.Prove that $3^{2n} - 1$ is a multiple of 8

TRIGONOMETRY.

34.Solve the equation: $5cos^2 2x = 3(1 + sin2x)$ fo $0 \leq x \leq 90$.

35.Simplify.$\frac{cos3x+cos5x}{sin5x-sin3x}$.

36.Solve the equation: $5sin2x - 10sin^2 x + 4 = 0$ for $-\pi \leq x \leq \pi$.

37.Prove that: $1 - tan^2 x + tan^4 x + \cdots = cos2x$.

38.Solve: $6cos2x + 7 = 7sin2x$ for $0^0 \leq x \leq 360^0$.

39.Prove that $sin2A + sin2B + sin2C = 4cosAcosBsinC$.

40.If $y = tanA + sinA$, $x = tanA - sinA$ are variable points of P, show that the locus of P is $(y^2 - x^2)^2 = 16xy$.

41.solve the equation $Cos3x - sin2x = sin3x - cos2x$ for $0 \leq x \leq 180$

42.Solve $8cos^4 x - 10cos^2 x = 0$ for $0 \leq x \leq 2\pi$.

43.solve the equation.$tan^2 x - sin^2 x = 1$ for $0^0 \leq x \leq 360$

44.Given that $cosA = \frac{3}{5}, cosB = \frac{12}{13}$ where A is reflex and B is acute, find the exact values of $\tan(A + B), cosec(A - B)$ without using calculators or tables.

45.Express $4cosx - 5sinx$ using $Rcos(x + \beta)$ and find the minimum and maximum value of the expression and where they occur.

46.find the minimum and maximum values of $\frac{1}{12cosx+4sinx+15}$.

47.Solve the equation: $4sin2x + 3cos2x = 3$ $0 \leq x \leq 180$

48.Show that $tan15 = 2-\sqrt{3}$.

49.Prove that $tan^{-1}\left(\frac{\sqrt{3}}{2}\right) + tan^{-1}\left(\frac{\sqrt{3}}{5}\right) = \frac{\pi}{3}$.

50.Solve for x in $tan^{-1}x + tan^{-1}(1-x) = tan^{-1}\left(\frac{9}{7}\right)$.

ANALYSIS

51.A rectangular field of 5000m^2 is to be fenced using a mesh. On one side of the field is a straight river. This side is not to be fenced. Find the dimension of the field that will minimize the wire mesh to be used.

52.Prove that $\frac{d(tan^{-1}x^x)}{dx} = \frac{(1+lnx)x^x}{1+x^{2x}}$.

53.An error of 3% is made in measuring the radius of the sphere. Find the percentage error in the volume.

54.If $y = e^{-x}cosx$, prove that $\frac{dy}{dx} = -\sqrt{2}\, e^{-x}cos\left(x-\frac{\pi}{4}\right)$.

55.use small changes to evaluate $\sqrt[3]{28}$,

56.find the volume generated when the area bounded by the curve $y = 5cos2x$, the x-axis and the ordinate $x = 0$ and $x = \frac{\pi}{4}$ is rotated about the x-axis through a complete revolution.

57.Evaluate $\int_0^{\frac{\pi}{2}} sin3xcosxdx$.

58.Sketch the curve $y = \frac{x^2-5x+6}{x-1}$ showing any asymptotes. Find the are enclosed by the curve, x-axis from $x = 2\ to\ x = 3$.

59.If $x = e^t$, $y = sinx$, show that $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

60.The radius of a sphere is increasing at a rate of $0.01cm/s$. find the rate at which surface area and volume increases when radius is $2cm$.

61.The curve $y = \frac{e^{2x}}{1+e^{2x}}$, show that gradient of the curve at $x = ln3$ is $\frac{9}{50}$.

62.Solve the following differential equation:

(i) $x\frac{dy}{dx} - y = xcos^2x$.

(ii) $cosx\frac{dy}{dx} + ysinx = 4x$

(iii) $x\frac{dy}{dx} = y(y-1)$.

(iv) $tanx\frac{dy}{dx} - y = sin^2x$.

(v) $(1-x)\frac{dy}{dx} = (1+x)y$, find $y(0) = 2$.

(vi) $xy\frac{dy}{dx} = 2x^2 + y^2$.

(vii) $\frac{dy}{dx} + y = e^{-x}cos\left(\frac{x}{2}\right)$.

(viii) $\frac{dy}{dx} = \frac{2x+y-2}{2x+y+1}$.

63.Integrate $\int_0^{\pi} e^x sinxdx$.

64.If $e^x y = sinx$, show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$.

65.Show that $\int_0^1 \frac{8x+6}{(x^2+1)(x+2)}dx = \pi + ln\frac{8}{9}$.

66.The curve $y = ax^3 + bx^2 + c$ has turning points at (0,4) and (-1,5). Determine the values of a, b and c. hence sketch the curve.

67.Integrate: $\int_1^2 \frac{8x+6\ dx}{(2x-1)^2(x+2)^2}$.

68.A rectangular figure with sides $8cm\ by\ 5cm$, equal sides of xcm are removed from each corner and the edge are turned up to make an open box of volume Vcm^3. Show that $V = 40x - 26x^2 + 4x^3$ and hence find the maximum possible volume and the value of x.

69.If $y = \frac{x+1}{\sqrt{x^2-1}}$, show that $\frac{dy}{dx} = \frac{-1}{(x+1)^{\frac{1}{2}}(x-1)^{\frac{3}{2}}}$

70.If $y = Ae^{3x} + Be^{-2x}$, show that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$.

71.Use small changes to find cos47^0

72.Given $y = \theta - cos\theta$, $x = sin\theta$. Show that $\frac{d^2y}{dx^2} = \frac{1+sin\theta}{cos^3\theta}$.

73.According to the newton's law of cooling, the rate at which the temperature of the body falls is proportional to the amount by which the temperature exceeds that of the surroundings. The temperature falls from 200^{0}c to 100^{0}c in 40minutes in a surrounding temperature of 10^{0}c. prove that after t minutes, the temperature T^{0}C of the body is given by $T = 10 + 190e^{-kt}\ where\ k = \frac{1}{40}ln\left(\frac{19}{9}\right)$. Calculate the time it will take the temperature to drop to 80C

74. Differentiate $y = ln\sqrt{\frac{3+2x}{3-2x}}$.

75. The rate of growth of a disease-causing virus increases at a rate proportional to the number of viruses present in the body if the number increases from 1000 to 2000 in I hour.
(i) How many days will be present after 1 and half-hours'
(ii) How long will it take the number of viruses in the body to be 4000.

76. If $y^2 - 2xy = 2x$, prove that $(x-y)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2$

77. Prove that $\int \frac{dx}{\sqrt{1+x^2}} = \ln(x + \sqrt{1+x^2})$.

78. Integrate $\int \frac{dx}{x^2\sqrt{1-x^2}}$.

79. Given that the curve $y = \frac{x(x-3)}{(x-1)(x-4)}$.
(i) show that the curve doesn't have turning points.
(ii) find the asymptotes for the curve and sketch the curve.

80. Show that $\int_0^{\frac{\pi}{2}} \frac{5dx}{4cosx+3sinx} = ln6$.

81. According to Newtons law of cooling, the rate of cooling of the body in air is proportional to the difference between the temperature of the body and that of the air. If the air temperature is kept at 25^0c and the body cools from 95^0c to 60^0c in 25minutes, in what further time will the body cools to 22^0c.

82. If $y = \sqrt{\frac{1+sinx}{1-sinx}}$, show that $\frac{dy}{dx} = \frac{1}{1-sinx}$

83. Differentiate $cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

84. A vessel containing water is in form of an inverted hollow cone with a semi vertical angle of 30^0.there is a small hole in the vertex of the cone and water is running out of the cone at a rate of $3cm^3/s$. find the rate at which the surface area in contact with the water is changing when there is $81\pi cm^3$ of water remaining in the cone.

85. The length of a rectangular block is three times its width. The total surface area of the block is $180cm^2$. Find its maximum value.

GEOMETRY.

86. Find the locus of a circle whose centre lies on the line $y = 3x - 1$ and passes through the point $(1,1)$ *and* $(2,-1)$.

87. Find the equation of locus of point which moves such that its distance from $D(4,5)$ is twice its distance from the origin.
Sketch the parabola $y^2 = 12(x-4)$, statebthe focus and the equation of the directrix.

88. Find the equation of the tangent to the circle $x^2 + y^2 + 2x - 2y - 8 = 0$ at (2,2).

89. Show that the line $y = mx + c$ touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then $c^2 = a^2m^2 + b^2$. Find the equation of the tangents to the ellipse $4x^2 + 9y^2 = 1$, which is perpendicular to $y = 2x + 3$.

90. If the normal at $P(ap^2, 2ap)$ to the parabola $y^2 = 4ax$ meets the curve again at $Q(aq^2, 2aq)$. Prove that $p^2 + pq + 2 = 0$.

91. Find the coordinates of the point of intersection of the circle. $x^2 + y^2 - 6x + 4y - 13 = 0$ and $x^2 + y^2 - 10x + 10y - 15 = 0$.

92. A conic is given by $x = 4cos\theta, y = 3sin\theta$. Show that the conic section is an ellipse and determine its eccentricity.

93. Show that the circles $x^2 + y^2 - 16x - 12y + 40 = 0$ *and* $x^2 + y^2 - 4y = 16$ are orthogonal.

94. Prove that the following pairs of the circle touch each other externally or internally. $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 - 8x + 12 = 0$.

95. Find the length of the tangent to the circle from the point (5,7) to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$.

96. Given that the line $y = mx + c$ is a tangent to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that $c^2 = a^2m^2 + b^2$. Hence determine the equation of the tangents at the point $(-3,3)$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

97. Find the coordinates of the circumscribing circle which passé through the points (1,2), (2,5) and (−3,4).

98. Calculate the equation of the tangents to the parabola $y^2 = 4x$ at point $(t^2, 2t)$. The tangent to the parabola at $P(p^2, 2p)$ and

$Q(q^2, 2q)$ meets on the line $y = 3$ Find the equation of the locus of midpoint of RS.

99. Find the equation of the circle passing through points (2,3) and (4,5) having the center on the line $y = 4x + 3$.

100. Show that the line $x - 2y + 12 = 0$ touches the circle $x^2 + y^2 - x - 31 = 0$ and find the coordinates of the point of contact.

101. Find the length of the latus rectum for the standard parabola $y^2 = 4ax$, and hence find the length of the latus rectum of $y^2 - 4y - 20 = 8x$.

102. Find the parametric equation of the circle $(x + 1)^2 + (y - 2)^2 = 9$

103. Prove that the line $y = x + 6$ cuts the parabola $y^2 = 32x$ at two distinct points and find these two points.

104. Find cartesian equation of the curve represented by $x = \frac{1+t}{1-t}$ and $y = \frac{t}{1-t}$.

VECTORS.

105. (a) show that the points (3,3,1) (8,7,4) and (11,4,5) are vertices of the triangle.
(b) given that (2,13, -5), (3, β, -3) and (6, -7, α) are collinear, find the values of α and β.

106. Given points A (-3, -3), D (9,5) and B such that D divides AB externally in the ratio of 4:3. Find the coordinates of B.

107. The line passes through the points A(4,6,3) and B(1,3,3). find the vector equation of the line containing such points and hence show that a point C(2,4,3) lies on the line.

108. Vectors $a = 2i - 2j - 2k$ and $b = i - 3j + 2k$ form two sides of the triangle, find its area.

109. show that the lines $r = i + 3j + k + \beta(-i - 2j + k)$ and $r = 2i - 3j + 4k + \alpha(3i - 2j + k)$ intersect.

110. The point (6, -9,5) lies on the line $\frac{x-a}{3} = \frac{y-5}{b} = \frac{z-c}{-4}$ which is parallel to the plane $3x + y + 4z = 3$. Find the values of a, b, c and the shortest distance between the line and the plane.

111. Show that A (4, -8, -13), B (3, -2, -3), C (3,1, -2) are vertices of the triangle.

112. Given that $A(3, -2, 5), B(9, 1, -1)$. Find the coordinates of C such that C divides AB in the ratio of $5: -3$. (i) externally (ii) internally.

113. find the ratios in which the line joining the point $A(-2,3,3), B(1,2,3)$ is divided externally by point P (7,0, -1)

114. Two points A and B have position $a = 5i + 4j + k, b = -i + j - 2k$. find the position vector of C such that $AC = 2CB$. (b)the angle between $r_1$and r_2 is $\cos^{-1}(\frac{4}{21})$. If $r_1 = 6i + 3j - 2k$ and $r_2 = -2i + \alpha j - 4k$. find the value of α.

115. Given $r = \begin{pmatrix} 5 \\ 3 \\ -5 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and $r = \frac{x-7}{-3} = \frac{y+1}{-2} = \frac{z+4}{-2}$ intersect, find the point of intersection. (b) find the cartesian equation of the plane through the point of intersection of the above lines and parallel to $x = \frac{y-2}{2} = \frac{z-2}{3}$.

116. find the line of intersection of the planes $2x + 3y + 4z = 1$ and $x + y + 3z = 0$.

117. The straight line l passes through the points with coordinates $(-5,3,6)$ and $(5,8,1)$. The plane p has equation $2x - y + 4z = 9$. Find the coordinates of the point of intersection between l and p.
(ii) Find the acute angle between l and p.

118. Find the perpendicular distance of the point (3, 0, 1) from the line $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z}{12}$.

119. Given $m = i - 3j + 3k, n = -i - 3j + 2k$. determine the equation of the plane containing the point (2,1, -1) and vectors m and n. hence find the angle the line $\frac{x-2}{4} = \frac{y}{3} = \frac{z-1}{2}$ makes with the plane above.

MATHEMATICS SEMINAR AT NABISUSA GIRLS SCHOOL

STATISTICS AND PROBABILITY

1. A random variable X takes the integer value x with $P(x)$ defined by, $P(X = 1) = P(X = 2) = P(X = 3) = kx^2$, $P(X = 4) = P(X = 5) = P(X = 6) = k(7 - x)^2$

Find the;

a) value of the constant k, hence sketch the graph of $f(x)$

b) $E(Y)$ and $Var(Y)$ where $Y = 4x - 2$

2. a) The chance that Moses wins a game is $\frac{1}{3}$. If he plays nine games in a row, what is the;

i) expected number of games,

ii) chance of winning at least two games.

b) At a bottle manufacturing factory, the new machine approximately makes 19% of the bottles that are damaged. If a random sample of 400 bottles is taken, find the probability that;

i) more than 81 bottles will be damaged,

ii) between 60 and 80 bottles inclusive will be damaged.

C) A sweet is randomly selected from a box A, which contains 3 green and 5 blue sweets. If a green sweet is selected, then a second sweet is selected from another box B that contains 2 red and 4 white sweets, otherwise a sweet is selected from box C contains 3 red and 2 white sweets. Find the probability that:

(i) The second sweet selected is red

(ii) A green sweet is selected from A given that the second sweet is red

3. The positive error made by a machine while manufacturing metal strips is a random variable which can take up any value up to 0.5 cm. it is known that the probability of the length being not more than y centimetres $(0 < y < 0.5)$ is equal to ky. Determine the;

a) value of k,

b) median positive error,

c) probability distribution function,

d) expected value of y,

e) standard deviation correct to 3.s.f.

4. On day one of the Coachella music festival, the height of the revellers can be modelled into a normal distribution of mean 1.75m and variance 0,0064m2. A draw is to be carried out and it is decided that one should have a height greater than 1.67m but less than 1.83m to participate.

a) Find the:

i) percentage of the people who qualify to take part in the draw.

ii) fraction that is rejected because they are too tall.

By day three of the event, the heights of the people present are normally distributed with mean, μ, and standard deviation, 0.085m. When the criteria used to select participants is not altered, 3.5% of the revellers are rejected because they are too short.

b) Find the:

i) value of μ,

ii) probability that a reveller whose height exceeds the mean qualifies to take part in the draw

5. The germination time of a certain species of beans is known to be normally distributed. In a given bath of these beans, 20% take more than 6 days to germinate and 10% take less than 4 days.

a) Determine the mean and standard deviation of the germination time.

b) Find the 99.5% confidence limits of the germination time.

6. The table below shows the consumer prices per unit of commodities A, B, C and D in 2015 and 2017, with corresponding weights.

Commodity	Price in Shs.		Weights
	2015	2017	
A	9000	11000	9
B	6000	7000	6
C	x	y	2
D	6000	8000	3

Given that the simple aggregate index and the cost-of-living index were 124 and 122.5 respectively in 2017 basing on 2015, find the values of x and y.

7. A discrete random variable Y has a $p.d.f$ given as;

$$f(y) = \begin{cases} ky & ; y = 1,2 \\ k(6-y) & ; y = 3,4 \\ k & ; y = 5,6 \end{cases}$$

where k is a constant. Determine the value of;

i) Value of k hence sketch f (Y) and F(Y)

ii) ii) E (Y)

8. Two tetrahedral dice both numbered 1 to 4are thrown. If one dice is un biased and the other one unbalanced such that a four is twice as likely as any other number to show, find the,

a) Probability that a sum of five is obtained

b) Probability that Sum of at most 4 is scored

Construct the probability distribution function for the sum of scores of the two tetrahedrons and find the;

i) Mean

ii) Standard deviation

9. A continuous random variable X has a probability distribution function defined by

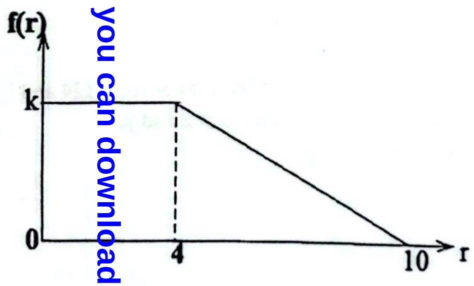
$$f(x)=\begin{cases} k(3-x); & 1<x<2 \\ k \quad ; & 2<x<3 \\ k(x-2); & 3<x<4 \\ 0 \quad ; & otherwise \end{cases}, \text{ where } k \text{ is a constant.}$$

Sketch the graph of $f(x)$ and find the value of k.

i) Determine the semi-interquartile range.

ii) Calculate $P(2.4 < X < 3.5)$.

10. A continuous random variable R has a probability density function (pdf), f(r) shown graphically below



a) Find the,

i) value of k

ii) expression of the p.d.f, f(r)

b) Determine the distribution function (c.d.f) of R and sketch it.

c) Calculate $P(3 \le R < 7)$

11. The table below shows the distribution of a random sample of marks of a group of candidates during an examination.

Marks	Cumulative Frequency
$0-<10$	10
$10-<20$	35
$20-<40$	65
$40-<60$	107
$60-<70$	123
$70-<95$	138

a) Calculate the; (i) Mean.

(ii) Standard deviation of the distribution.

b) If the sample was taken from a population which is approximately normally distributed, determine the 99.5% confidence limits for the population mean mark, correct to two decimal places.

c) Estimate the mode.

12. a) The box contains **g** green and **d** red pens.one pen is drawn at random, when it's put back in the box, **k** additional pens of the same colour are put with it. now suppose that we draw another pen show that the probability that first pen drawn was green given that the second pen drawn was red is $\frac{g}{g+d+k}$.

b) At Nabisunsa , the students court may give one of the three verdicts **"guilty","** **not guilty"** and **"not proven"**. Of all cases tried by the court this year, 70% of the verdicts are **guilty** 20% are **not guilty** and 10% are **not proven.** When the courts verdict is guilty, not guilty and not proven, the probabilities that the accused is really innocent are 0.10,0.80 and 0.20 respectively. calculate the probability that an innocent will be found guilty by the court

13. In many schools' teachers complain about typing errors, a test was designed to investigate the relationship between typing speed and errors made 12 typists were selected, the table below shows their speeds (y) and the number of errors (x)

Complied by shil@swalehe

No. of errors	12	24	20	10	32	30	28	15	18	40	27	35
Speed(s)	130	136	124	120	153	160	155	142	145	172	140	157

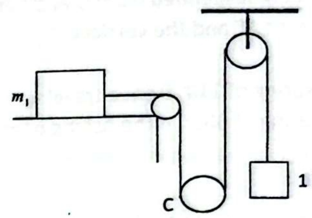
(i) Plot the data on a scatter diagram
(ii) Draw a line of best fit on your diagram and comment on the likely association between speed and errors made
(iii) By giving rank 1 to the fastest typist and the typist with least errors, rank the data above and use them to calculate the rank correlation coefficient, test the significance at 5 % level.

MECHANICS

14. A particle has an initial position vector of $(4i + 3j - 7k)$m . The particle moves with a constant velocity of $6ms^{-1}$ parallel to $2i - j + 2k$. Find the position vector of the particle 3s later. Hence find how far the particle is from the origin.
15. A particle A of mass 2 kg moves under the action of a constant force $(2i + 4j + 3k)$N At t = 0s the particle is stationary and at a point with position vector $(4i - 10j + 7k)$m . Find the position vector of the particle at time $t = 5s$.
16. Forces of magnitude 2N, 4N, 6N, 1N, and 5N act along the sides AB, BC, DC, ED and EA respectively of a regular pentagon of length 1.5m. Taking AB as the reference x axis, find the:
    a) Magnitude of the resultant force and its direction
    b) Line of action by taking moments about point A. Hence determine where the line of action crosses AB
17. A motorcar starting from rest and moving with uniform acceleration goes $9.5m$ in the 10th second find the,
    i) Acceleration of the car
    ii) Velocity at end of $10s$
18. A non-uniform ladder AB of length 10m, weighing 5W and centre of mass 4m from A rest in a vertical plane with end B against a rough vertical wall and the end A against a rough horizontal surface. The angle between the ladder and the horizontal is 50^0 and the coefficient of friction at each end is $\frac{1}{4}$ *and* $\frac{1}{2}$ and respectively.
    a) A man of weight 13W begins to ascend the ladder from the foot, find how far he will climb before the ladder slips?

    b) If a horizontal inextensible string is the attached from end A to the base of the wall, find the tension in the string when the man climbed the ladder to end B.

19. A particle of mass m_1 *kg* is at rest on a smooth horizontal table. It is attached to a light inextensible string. The string, after passing over a small fixed pulley at the edge of the table, passes under a small moveable pulley C of mass m_2 *kg* . The string then passes over a smooth fixed pulley and supports a mass of 1 kg.



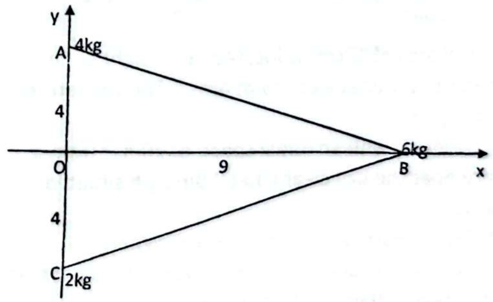
The system is released from rest.

(a) Show that the tension in the string is $T = \dfrac{3m_1m_2g}{4m_1 + m_2 + m_1m_2}$.

(b) The pulley C will remain at rest if $\dfrac{2}{m_2} - \dfrac{1}{m_1} = k$. Find the value of k

In the pulley system shown below, A is a fixed pulley and pulley B has a mass of 40 kg. A particle of mass 50 kg is attached to the free end of the string.

20. The figure below shows a triangular laminar ABC.



The co-ordinates of A, B and C are (0, 4), (9, 0) and (0, -4) respectively. If the particles of mass 4kg, 6kg and 2kg are attached at A, B and C respectively.

(i) Calculate the co-ordinates of the centre of mass of the three particles.

(ii) The centre of mass of the combined system consisting of the three particles and the lamina has coordinates (4, λ). If lamina ABC is uniform and of mass m kg, calculate the value of m and λ.

(iii) The combined system is now freely suspended from O and hangs at rest, determine the angle between AC and the vertical.

21. At 9:00am, a fishing boat is 10km on a bearing of 110° from a traveler, travelling with a speed of $8kmh^{-1}$ on a bearing of 060°. If the fishing boat has a top speed of $6kmh^{-1}$, find the;

(a) route of the fishing boat if it is to be as close to the traveler as possible

(b) distance between the two boats at this point and the time at which it will occur

22. (a)The position vector of one particle relative to another is $(2t-5)\mathbf{i} + (10-t)\mathbf{j}$ After t seconds. Determine the respective relative velocity, hence or otherwise calculate the shortest distance between the particles.

(b) A ship traveling at $40kmh^{-1}$ due to 080^0E is initially at a point $25km$ north wet of a patrol vessel. The patrol is capable of reaching a maximum speed of $30kmh^{-1}$. Show that the patrol vessel can take two courses in order to intercept the ship, and determine the difference in the times of interception

23. a) A horse pipe of cross-sectional area of $12cm^2$ is located at a height of 6m above the ground draws and issues water at a speed of $4ms^{-1}$. find the rate at which water is oozing out of the pipe.

b) A tennis ball is served horizontally with an initial speed of $20ms^{-1}$ from a height of 3m. by what distance does the ball clear a net 0.9m high situated 12m horizontally from the server.

c) A particle is moving with Simple Harmonic Motion (SHM). When the particle is 15 m from the equilibrium, its speed is 6 m s^{-1}. When the particle is 13 m from the equilibrium, its speed is 9 m s^{-1}. Find the amplitude of the motion.

24. A force $24t\mathbf{i} - 12\mathbf{j}$ Newtons acts on a particle of mass 2kg initially at rest at point (-4,3). Find the;

a) position vector of the particle after t seconds.

b) work done by the force in the time interval $t = 1$ to $t = 2$ seconds.

25. A tennis player hits a ball at a point O, which is at a height of 2m above the ground and at a horizontal distance 4m from the net, the initial speed being in a direction of 45° above the horizontal. If the ball just clears the net which is 1m high, (a) Show that the equation of path of the ball is $16y = 16x - 5x^2$.

(b) Calculate the;

(i) Distance from the net at which the ball strikes the ground

(ii) Magnitude and direction of the velocity with which the ball strikes the ground. (Use $g = 10ms^{-2}$)

26. A pile driver of mass 1200kgs falls freely from a height of 3.6m and strikes without rebounding a pile of mass 800kgs. The blow drives the pile a distance of 36cm into the ground.

Find the;

i) Resistance of the ground

ii) Time for which the pile is in motion.

27. (a) A lorry of mass 800kg is pulling a trailer of mass 200kg up a hill of 1 in 14 When the total force of 1KN is exerted by the engine, the car and the trailer move up the hill at a steady speed. Find the total frictional resistance to the motion of the car and the trailer during the motion.

(b) When the car and the trailer are traveling at a speed of $10m^{-1}$ up the hill, the power exerted by the engine is instantaneously changed to 2kw. Calculate the:

(i) Instantaneous acceleration.

(ii) Instantaneous tension in the coupling between the trailer and the car given that the total frictional resistance on the trailer is 70N.

28. a) Three points A, B and C lie along a straight line. The distance between A and B is 95m, and B and C is 80m. A particle travelling with a constant acceleration along the straight line towards C passes point A with velocity u. If it covers the distances between A and B in 5 seconds and between B and C in 2 seconds, find

i) the velocity, u and acceleration of the particle.

ii) the distance beyond C, the particle covers in the next 3 seconds.

(b) A stone is projected vertically upwards with a speed $20ms^{-1}$ from a tower of height 25m. find the;

i) time taken for the stone to reach maximum height.

ii) velocity with which the stone hits the ground.

NUMERICAL METHODS

29. Show graphically the roots of the equation $x^5 + 1 = 3x^2$,hence use linear interpolation to find the least root of the equation correct to two decimal places.

30. Show that the Newton's Raphson's formula for finding the natural logarithm of a number z is $x_{k+1} = \frac{z + e^{x_k}(x_k - 1)}{e^{x_k}}$; $k = 0, 1, 2, \ldots$ ....hence draw a flow chart;

(i) Reads the initial approximation x_0 and reads the number z

(ii) Computes and prints the root and z after three iterations to three decimal places; perform a dry run for the flow chart taking $z = 30$ *and* $x_0 = 3.2$.

31. Show that the Newton's Raphson's formula for finding the natural logarithm of a number z is $x_{k+1} = \frac{z+e^{x_k}(x_k-1)}{e^{x_k}}$; $k = 0, 1, 2, \ldots$ ....hence draw a flow chart;

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32. a) The height of the top of a ladder of length l resting against a vertical wall making an angle of θ^0 with the horizontal is given by $h = l \sin\theta$.

i) Show that the maximum relative error made in estimating the height h is given by $\left|\frac{\Delta l}{l}\right| + \left|\frac{\Delta\theta}{\tan\theta}\right|$, where Δl and $\Delta\theta$ are the respective errors in l and θ.

ii) Find the maximum relative error in h if l and θ are measured to be $3.96m$ and 59^0 respectively.

b) The length and width of a rectangle are measured as 4.5m and 2.4m with percentage errors of 5% and 2% respectively.
Determine the,

i) Range within which its area lies. ii) Maximum possible error made in estimating its perimeter.

33. a) Given the numbers A and B with A = 7.35 B = -8.214 measured to the nearest decimal places indicted. Determine the absolute error in A/B, hence the limits within which the quotient A/B lies correct to 3 decimal places.

b) Given that $y = \theta\cos\theta$ where θ is measured with a maximum possible error of 5%, find, for θ = 120°,

(i) the maximum error in y,

(ii) the interval within which the values of y lie, correct to 4 decimal places

34. In the table below, is part of an extract of cot x°.

$X = 70^0$	12ꞌ	15ꞌ	30ꞌ	38ꞌ	45ꞌ
cot x	0.3600	0.3590	0.3541	0.3515	0.3492

Use linear interpolation or extrapolation to estimate the;

(i) value of $\cot 70^0\,12ꞌ$

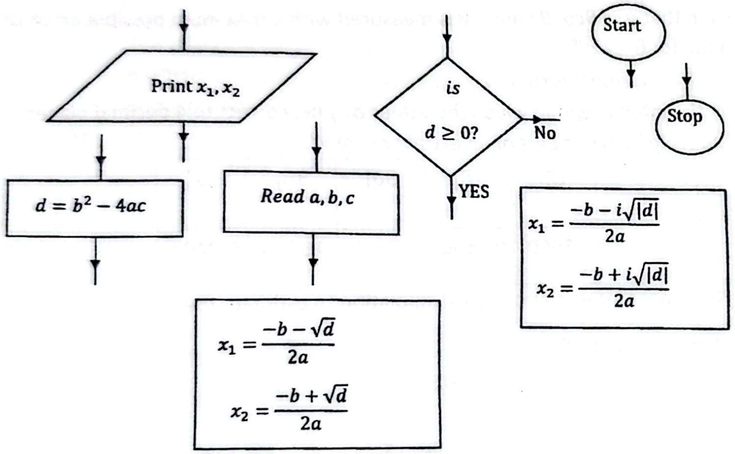
(ii) Angle whose cotangent is 0.3394

35. (a) Use the trapezium rule with equal width of $\frac{\pi}{6}$ to estimate $\int_0^{\frac{2\pi}{3}} (x - \sin x)\,dx$. Give your answer correct to 3 decimal places.

(c) Determine the percentage error made in the estimation

36. Show that the simplest formula based on Newton Raphson method, for finding the natural logarithm of a number A is $x_n - 1 + Ae^{-x_n}$. Hence, taking the initial approximation, $x_0 = 1.9$, use the formula twice to find $\ln 7$.

35. The method for solving the quadratic equation $ax^2 + bx + c = 0$ is described in the following parts of the flow chart.



(i) By re-arranging the given parts, draw a flow chart that shows the algorithm for the described method.

(ii) Perform a dry run for $x^2 - 4x + 13 = 0$, and state the roots of the equation.

36. Construct a flow chart that can be used to find the mean of the cubes of the first *n* counting numbers. Perform a dry run for *n = 10*.

END