P425/1 PURE MATHEMATICS Paper 1 April 2025 3 Hours

SET 5

GRANT K EXAMS 2025

Uganda Advanced Certificate of Education

PURE MATHEMATICS

PAPER 1

3 Hours

INSTRUCTIONS:

- Attempt all the **eight** questions in section **A** and any **five** from section **B**.
- All working must be clearly shown.
- Mathematical tables with a list of formulae and squared paper are provided.
- Silent, simple non programmable scientific calculators and a list of formulae may be used.
- State the degree of accuracy at the end of each answer using **CAL** for calculator and **TAB** for tables.

SECTION A (40 Marks) Attempt ALL questions in this section.

- 1. Solve the inequality $\frac{1+x}{4+x} \ge \frac{5-2x}{x}$ (5marks)
- 2. Evaluate $\int_{3}^{4} \frac{1}{x^2 3x + 2} dx$

3.

Solve the equation $2tan\theta + sin2\theta sec\theta = 1 + sec\theta$ for $0 \le \theta \le 2\pi$. (5marks)

(5marks)

- 4. The line 5x-2y + 8 = 0 is a tangent to the circle with centre at (-2, 3). Find the equation of the circle. (5marks)
- 5. Expand $(25 2x)^{\frac{1}{2}}$ in ascending powers of x up to the term in x^3 . Hence by taking x=1, obtain the value of $\sqrt{23}$ correct to four significant figures. (5marks)

6. If
$$y = e^{2x} sin 2x$$
, show that $\frac{d^2 y}{dx^2} = 8(2e^{2x} cos^2 x - 1)$. (5marks)

7. The position vectors of the points P and Q are $3\underline{i} - \underline{j} + 2\underline{k}$ and $2\underline{i} + 2\underline{j} + 3\underline{k}$ respectively. Find the acute angle between PQ and the line; $1 - x = \frac{y-3}{2} = \frac{4-x}{2}$ (5marks)

8. Solve the differential equation, $\left(\frac{dy}{dx}\right)^3 = e^{(x-3y)}$. Given that y (6) = 0. (5marks)

SECTION B (60MARKS) Attempt ONLY 5 questions in this section.

a) Show that; $(xy) = \frac{1}{2}x + \frac{1}{2}y$. Hence or otherwise, solve the simultaneous equations. $(xy) = \frac{7}{2}$

$$\frac{x}{y} = -8 \tag{7marks}$$

b) Solve the equation $2^{(2+2x)} + 3 \cdot 2^x - 1 = 0.$ (5marks)

- a) Find x, if $\cos^{-1}\left(\frac{x}{2}\right) = \frac{5\pi}{6}$. (5marks)
 - b) Express $5sin\phi + 12cos\phi$ in the form $r sin(\phi + a)$, giving the value of r and a, hence find $5sin\phi + 12cos\phi = 7$. (7marrks)

9.

10.

11. a) Differentiate with respect to x. i) x^{x} ii) $tan^{-1}\left(\frac{1-x}{1+x}\right)$, simplify your answers (8marks) b) if $y = e^{4x}cos3x$, show that $\frac{d^{2}y}{dx^{2}} - 8\frac{dy}{dx} + 25y = 0$. (4marks)

12. a) Show that the line
$$\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$$
 and the plane $\underline{r} \cdot (4 - 1 - 3) = 4$ are parallel and find the perpendicular distance of the line from the plane. (6marks)

b) Find the equation of the plane passing through the origin and parallel to the lines' $\frac{x+2}{3} = \frac{y-1}{4} = \frac{z+1}{5} \text{ and } \frac{x-3}{4} = \frac{y-2}{-5} = \frac{z+1}{1}.$ (6marks)

3. a) Solve the differential equation

$$x^2 \frac{dy}{dx} = y(y + x)$$
; Given that $y(4) = 6$. (4marks)

- b) A certain game park was found to have 100 lions. Given that the lions die at a rate proportional to the number of lions present and the initial death rate is 5 lions per year.
- i) Form a differential equation and solve it.
- ii) How many lions will be in the park after six years? (8marks)
- 14. a) Given that $Z = \cos \cos \phi + i \sin \phi$, where $\phi \neq \pi$, show that $\frac{2}{1+z} = 1 i \tan(\frac{1}{2}\phi)$. (6marks)
 - b) The polynomial $P(z) = z^4 3z^3 + 7z^2 + 21z 26$ has 2 + 3i as one of the roots. Find the other three roots of the equation P(z) = 0. (6marks)

15. a) Work out
$$\int \frac{dx}{e^x - 1} dx$$
. (5marks)

b) The area bounded by the curve y = x(x - 4), and the x-axis is rotated about the x-axis through a $\frac{1}{2}$ -turn. Find the volume of the solid generated. (7marks)

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- Find an equation of the circle that passes through the points. a) A(-1, 4), B(2, 5) and C(0, 1).(5marks)
 - The line x + y = c is a tangent to the circle b) $x^{2} + y^{2} - 4y + 2 = 0$. Find the coordinates of the points of contact of the tangent for each value of C. (7marks)

