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S6 MOCK EXAMINATIONS JULY 2005
P425/2 APPLIED MATHEMATICS
PAPER 2
3 HOURS

Instructions:
Attempt all questions in section A and **NOT** more than **FIVE** in section B

SECTION (40 MARKS)

1. A and B are events such that $P(A/B) = {}^{1}/_{5}$, $P(\bar{A}nB) = {}^{3}/_{10}$

   and $P(B/\bar{A}) = {}^{2}/_{5}$.

   Find:
   (i) P(AnB)
   (ii) P(AUB)

   (5 marks)

2. Study the table below

x	y	2y	3y
f(x)	0.16	0.48	0.64

   Using linear approximation find
   (i) f(½ y)
   (ii) n such that f(ny) = 0.5

   (5 marks)

3. A particle executing SHM starts from rest and when it has covered distances of 2m, 4m its speeds are $12ms^{-1}$, $16ms^{-1}$ respectively. Calculate the shortest time taken between the two points (5 marks)

4. X is a distribution where mean μ =16 and variance, σ^2 = 25, samples of size 64 are picked at random from the distribution. Find

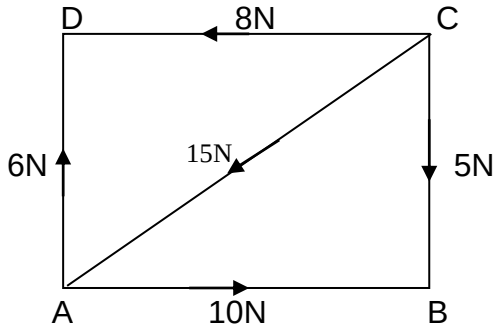
   (i) the mean of $\mu\bar{X}$
   (ii) the variance of , $\sigma^2\,\bar{X}$

   (iii) $p(\bar{X}>17)$

   where $\bar{X}$ is the distribution of the sample means (5 marks)

5. Show that the equation 2x = cos2x has a root between 0 and 1
   If $\pi/4$ is a first approximation, show that the second approximation using the Newton – Raphson method is $\pi/8$ (5 marks)

6.



ABCD is a rectangle in which $\overline{AB}$ = 8m, $\overline{BC}$ = 6m. Find the equation of the line of action of the resultant of the forces indicated. State the point at which the line meets $\overline{AB}$ (5 marks) .

7.

Life expectancy	0 – <5	5 – <15	15 – <35	35 – <45	45 – <55	55 - <80
No of people	16	24	36	40	80	14

The life expectancy in years of 210 people was recorded as shown in the table. Calculate the:

(i) median life expectancy of the group
(ii) the number of people whose life expectancy was below 40 years.
(5 marks)

8. A uniform rod AB o weight λP Newton is smoothly hinged to a vertical wall at A. A force P Newton applied at end B perpendicular to the rod keeps the rod in equilibrium making 30^o with the downward vertical.
Find the
(i) value of λ
(ii) the magnitude of the reaction at the hinge in terms of P.

SECTION B (60 MARKS)

9. (a) Given that X = 2.4, Y = 3.24 and Z = 0.6512, are rounded off
(i) find the limits within which the exact value of
$X + \frac{Y}{Z}$ lies
(ii) Calculate the maximum percentage error in $\frac{X^2Z}{\sqrt{Y}}$

(b) Using 6 ordinates evaluate $\int_4^5 \log_{10}(x - 2)\, dx$ to 2 decimal places using the trapezium rule. Compute the absolute error in your answer.

10. (a) The probabilities a man reads Newvision, Monitor, the Observer newspapers are 0.6, 0.4, 0.2 respectively. Calculate the probability that he reads
    (i) at least one paper
    (ii) New Vision or the Monitor (5 marks)

    (b) A regular tetrahedron is numbered 1 to 4. When it is thrown the probability that a number occurs at the base is proportional to that number.
    Find
    (i) the expected number that will occur
    (ii) the least number of throws of the tetrahedron if the probability that a 4 occurs at least once exceeds 0.95

11. AB and BC are uniform rods of length 6m, 8m and weight W and 3W N respectively smoothly jointed at B. The ends A and C of the rods are in contact with a smooth horizontal surface and connected by an inelastic sting. The system is in equilibrium in a vertical plane with angle ABC = 90^o. Calculate
    (i) the normal reactions at A and C
    (ii) the magnitude and direction of the reaction at the hinge
    (12 marks)

12. (i) Show that the root of the equation $e^{1/x} = x$ lies between 1.7 and 1.8.

    (ii) use linear interpolation to find the first approximation to the root of the equation

    (iii) Prove that the Newton –Raphson formula for approximating the root of the equation above is $X_{n+1} = \frac{1 + X_n}{1 + \ln X_n}$, n = 0,1,2,….
    Hence construct a flow chart for finding the root correct to 3 decimal places

    (iv) Perform a dry run for your flow chart using the first approximation obtained in step (ii) above. (12 marks)

13. (a) X is a continuous random variable whose pdf is

$$f(x) = \begin{cases} K\cos 2x ; & 0 \leq x \leq \pi/4 \\ 0 \quad \text{elsewhere.} \end{cases}$$

    Find the value of k, and the median of X. (5 marks)

    (b) The diameters of soccer balls are normally distributed with mean 15 cm and standard deviation 2.5 cm. Balls of diameters less than 12 cm and greater than 20cm are rejected by the World soccer body; FIFA.
    What is the probability that
    (i) exactly 4 out of 10 such balls are rejected?
    (ii) at most 20 out of 100 balls will be rejected? (7 marks)

14. A force **F** =(t^2 + 4) **i** + 2t **j** Newtons acts on a particle for ts at which instant the velocity of the particle is **v** = 4t **i** + (12 – t^2) **j**$ms^{-1;}$ timing commences when t = 0
    (i) calculate the power developed at t = 1s
    (ii) determine the speed of the particle at t = 3s; and the angle the direction of the particle makes with the x – axis at this instant.
    (iii) Write down the expression for the acceleration of the particle at any time t.
    (iv) Show that there are two instants from commencement of timing at which the velocity and acceleration vectors are perpendicular; and determine the average acceleration of the particle between these instants.

15.

Heights 1cm	No of students
50 – 69	9
70 – 89	13
90 – 109	21
110 – 129	37
130 – 149	12
150 – 169	6
170 – 189	2

A random sample of 100 students was taken from a school and their heights measured and represented as shown in the table.

(a) Construct a histogram for this data , and use it to estimate
    (i) the model height
    (ii) the median height

(b) Using an assumed mean of 119.5 calculate
    (i) the mean height
    (ii) the standard deviation of the heights

(c) Assuming the heights of the students are normally distributed compute a 97% confidence interval for the mean height of the whole school

(12 marks)

16. One end of an elastic string of natural length l is attached to point A at the ceiling. The other end carries a particle of mass mkg. The particle initially at A is released from rest to fall vertically downwards; and thereby undergoing an extension e_o at equilibrium
    (a) Show that the maximum extension e in the string is given as
    $e = (1 + \beta)e_o$, where $\beta = (1 + {}^{2l}/_{e_o})^{\frac{1}{2}}$

    (b) Given that $e_o : l = 1:4$ determine
        (i) the modulus of elasticity of the string
        (ii) the ratio of the energy stored in the string at equilibrium to that stored at maximum extension.
        (iii) The maximum speed of the particle in its motion, in terms of g and l.

(12 marks)

END