PRINCIPAL MATH TOPICAL PRACTICE ITEMS

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TOPIC 1: NUMERICAL CONCEPTS

Item 1

Nakato is a mobile money agent in her village, Mukono. When she started in January 2024, she had 50 regular customers. She observed that her customer base seemed to grow exponentially each month. By the end of March 2024 (after 3 months), she had 135 regular customers. She wants to predict her customer growth to plan for liquidity (cash and e-float) and potentially hire an assistant. Assume the growth follows the model $N = N_0 \times K^t$, where N is the number of customers after t months, N_0 is the initial number of customers, and K is the monthly growth factor.

Tasks:

a) Help Nakato in determining her monthly growth factor, K. Express your answer to 3 significant figures.

b) Determine the number of customers Nakato can expect by the end of December 2024 (after 12 months) if this growth rate continues.

c) Nakato estimates she needs UGX 10,000 in float per regular customer per month. Using the predicted number of customers for December 2024, calculate the total float she would need. Express this amount using index notation in terms of powers of 10.

Item 2

Mr. Okello is weaving a traditional Ugandan mat (ekikeeka) with intricate geometric patterns. One key element involves fitting square tiles made of dyed reeds into a rectangular border. The side length of each square tile needs to be exactly $(\sqrt{5} - \sqrt{2})cm$ for the pattern to align perfectly. The rectangular border has a length of $(10\sqrt{5} + 5\sqrt{2})cm$ and a width of $(8\sqrt{5} - 4\sqrt{2})\ cm$.

Tasks:

a) Calculate the exact area of one square tile. Express your answer in the simplest surd form $a + b\sqrt{c}$.

b) Determine the exact area of the rectangular border. Express your answer in the simplest surd form.

c) If Mr. Okello wants to fit as many *whole* square tiles as possible within the border without overlapping, estimate the maximum number of tiles he can fit. Justify your answer.

Item 3

A local environmental group in Jinja is studying the population growth of a specific fish species in a protected section of the Nile River. Their initial estimate in 2020 was 1,200 fish. They believe the population **P** after $\boldsymbol{t}$ years can be modelled by $P(t) = 1200 \times (1.15)^t$. However, another model proposed is based on logarithms: $\log_{10} P = \log_{10} 1200 + t \log_{10} 1.15$.

Tasks:

a) Using the index model $P(t) = 1200 \times (1.15)^t$, calculate the predicted fish population in the year 2025.

b) Using the logarithmic model, show that it is equivalent to the index model.

c)

d) The group wants to know when the fish population is predicted to reach 5,000. Using logarithms and the model $P(t) = 1200 \times (1.15)^t$, determine the approximate number of years (t) it will take.

TOPIC 2: EQUATIONS AND INEQUALITIES

Item 4

Mrs. Nabukalu, a farmer in Masaka, finds that the yield of her maize crop (in bags per acre), Y, depends on the amount of a specific fertilizer x used (in kg per acre). The relationship is modelled by the quadratic equation: $Y(x) = -0.5x^2 + 20x + 50$. She wants to maximize her yield but also knows the fertilizer costs money.

Tasks:

a) Help Mrs. Nabukalu to determine the amount of fertilizer (x) that will give her the maximum maize yield.

b) Calculate the maximum possible yield in bags per acre.

c) If the cost of the fertilizer is UGX 1,500 per kg. Mrs. Nabukalu wants the yield to be at least 200 bags per acre. Formulate a quadratic inequality to represent this situation.

d) By solving the inequality in part (c) above determine the range of fertilizer amounts (in kg per acre) she can use to achieve a yield of at least 200 bags per acre.

Item 5

A community group in Gulu is managing the costs for drilling three boreholes (A, B, and C). The total cost was UGX 25,000,000. The cost of Borehole B was UGX 1,000,000 less than Borehole A. The combined cost of Boreholes A and C was three times the cost of Borehole B. Let the costs of drilling boreholes A, B, and C be $a, b,$ and c (in UGX) respectively.

Tasks:

a) Formulate a system of three linear simultaneous equations representing the information given.

b) By Using Row reduction, calculate the individual cost of drilling each borehole (a, b, and c).

c) If the cost per meter drilled was UGX 250,000 for all boreholes, determine the depth of Borehole A.

Item 6

A school in Mbarara wants to create a rectangular vegetable garden. They have 80 meters of fencing available. They want the area of the garden to be greater than 300 square meters to grow enough vegetables for the school lunch program. Let the length of the garden be L meters and the width be W meters.

Tasks:

a) Express the perimeter of the garden in terms of L and W and form an equation using the available fencing.

b) Using your equation in a) above formulate the area A of the garden purely in terms of L.

c) Formulate a quadratic inequality representing the condition that the area must be greater than 300 square meters and solve it to determine the possible range of values for the length (L) of the garden that satisfies both the fencing constraint and the area requirement.

TOPIC 4 COORDINATE GEOMETRY 1

Item 7

Mr. Kato owns a rectangular farm near Fort Portal. On a map grid, the corners of his main plot are at $A(1,2), B(9,2), C(9,8)$, and $D(1,8)$. He plans to install two straight irrigation pipes. *Pipe 1* will run from corner A to corner C. *Pipe 2* will run from the midpoint of side AB to the midpoint of side CD. A water source is located at point W (5, 5).

Tasks:

a) Determine the coordinates of the midpoints of sides AB and CD.

b) Formulate the equation of the line representing Pipe 1 (line AC) and the equation of the line representing Pipe 2.

c) Determine the shortest distance from the water source W (5, 5) to Pipe 1 (line AC). Will Pipe 1 pass directly through the water source? Justify your answer.

Item 8

A new road (Road 1) is being constructed in Kampala, represented by the equation $y = 2x + 3$. It will intersect an existing road (Road 2), represented by the equation $3x + 2y = 12$. A traffic light needs to be installed at the intersection point. Another planned road (Road 3) needs to be parallel to Road 1 and pass through the point $P(4,1)$. A fourth road (Road 4) must be perpendicular to Road 2 and pass through the same point $P(4,1)$.

Tasks:

a) Calculate the coordinates of the intersection point of Road 1 and Road 2 where the traffic light will be placed.

b) Determine the equation of the line representing the planned Road 3 and Road 4.

c) Calculate the acute angle between Road 1 and Road 2 at their intersection point. Give your answer in degrees.

Item 9

Three villages, A, B, and C, are located on a map grid at coordinates A(2, 1), B(8, 3), and C(4, 7). A new health centre needs to be built such that it is equidistant from villages A and B. It must also lie on the line that passes directly between village C and the midpoint of the line segment connecting A and B.

Tasks:

a) Determine the coordinates of the midpoint M of the line segment connecting villages A and B.

b) Formulate the equation of the perpendicular bisector of the line segment AB. (This line represents all points equidistant from A and B).

c) Find the equation of the line passing through village C (4, 7) and the midpoint M calculated in Task 1.

d) Calculate the coordinates where the two lines found in b) and c) intersect. This point represents the ideal location for the health centre. Justify why this location satisfies both conditions.

TOPIC 5: PARTIAL FRACTIONS

Item 10

An Engineer in a chemical engineering plant in Namanve, want to use a chemical with concentration $C(t)$ of a product over time $\boldsymbol{t}$, which is modelled by complex rational functions. Suppose the rate of change of concentration involves the expression: $f(t)\ = \frac{5t+3}{(t+1)(t+2)}$. To analyse the long-term behaviour but doesn't know the appropriate techniques to use.

Tasks:

a) Help the engineer to identify the type of factors in the denominator of $f(t)$.

b) Express $f(t)$ as the sum of its partial fractions.

Item 11

An electrical engineering student at Makerere University is analysing a signal whose behaviour over time x is related to the function $g(x)\ = \frac{2x^2+x-1}{x(x-1)^2}$. This expression needs to be broken down for further analysis.

Tasks:

a) Set up the appropriate form for the partial fraction decomposition of $g(x)$.

b) Determine the values of the unknown constants in the partial fraction decomposition.

c) Write the final partial fraction decomposition of $g(x)$.

Item 12

An economist is studying the relationship between investment I and national income Y. The relationship involves a complex function where a particular term is given by $h(Y) = \frac{Y^3 + 2Y^2 - Y + 5}{Y^2 + Y - 2}$. Before proceeding with the economic analysis, the economist needs to simplify this expression.

Tasks:

Help the economist to;

a) Identify h(Y) as a proper or improper rational function. Justify your answer.

b) express h(Y) as the sum of a polynomial and a proper rational fraction.

c) Take the proper rational fraction part obtained in b) and decompose it into its partial fractions.

d) Combine the results from b) and c) to write the complete simplified expression for h(Y).

TOPIC 5: TRIGONOMETRY

Item 13

A surveyor is mapping a triangular piece of land in the hilly region of Kabale. The vertices are marked as P, Q, and R. The distance PQ is measured as 120 meters, and the distance PR is 150 meters. The angle at P, $\angle QPR$, is measured as $75°$. The surveyor needs to find the length of the third side QR and the area of the land.

Tasks:

a) Determine the length of the side QR to the nearest meter and angle $\angle PQR$ to 1 decimal place.

b) Calculate the area of the triangular piece of land PQR.

c) If $\angle QPR$ was actually m easured as $(45° + 30°)$, apply an appropriate formula find the exact value of $\cos 75°$.

Item 14

An architect is designing a symmetrical roof truss for a community hall in Lira. The truss is shaped like an isosceles triangle ABC, with AB = AC. The base BC has a length of 16 meters. The angle at the apex A, $\angle BAC$, needs to be determined such that the height (altitude from A to BC) is exactly 6 meters. Let M be the midpoint of BC.

Tasks:

a) Consider the right-angled triangle AMB. Calculate the length of the side AB

b) In triangle AMB, determine the value of $\tan(\angle ABM)$ and hence find $\angle ABM$ in degrees.

c) determine the measure of \angle BAM hence calculate the angle at the apex, $\angle BAC$.

Item 15

A fishing boat leaves Kasenyi landing site (Point K) and travels 15 km on a bearing of $060°$ to reach Point A. From Point A, it then travels 20 km on a bearing of $135°$ to reach Point B. The boat captain now wants to know the direct distance and bearing from Kasenyi (K) back to Point B.

Tasks:

a) Help the captain to map the journey on a diagram, showing the points K, A, B, and their bearings.

b) By applying cosine rule calculate the direct distance KB, correct to one decimal place.

c) By applying sine rule calculate the angle $\angle AKB$. Hence, determine the bearing of Kasenyi (K) from Point B.

TOPIC 6: DESCRIPTIVE STATISTICS

Item 16

A cooperative society of farmers in Luwero recorded the cassava yield (in tonnes per hectare) from 50 small plots. The data is grouped as follows:

Yield (Tonnes/Hectare)	**Number of Plots (Frequency)**
5 - < 10	6
10 - < 15	10
15 - < 20	15
20 - < 25	11
25 - < 35	8

Tasks:

a) Construct a histogram to represent this data.

b) Using the histogram, estimate the modal yield of cassava per hectare.

c) Calculate an estimate of the mean yield and the standard deviation of the yield for these plots.

Item 17

The scores of 80 Senior Five students in a Mathematics mock exam at a school in Arua are summarised in the following cumulative frequency table:

Score (x)	Cumulative Frequency
$x \leq 20$	5
$x \leq 30$	15
$x \leq 40$	35
$x \leq 50$	55
$x \leq 60$	70
$x \leq 70$	77
$x \leq 80$	80

Tasks:

a) Represent the data on a cumulative frequency curve (ogive) to represent this data and use it to estimate:

   i) The median score.

   ii) The interquartile range of the scores.

   iii) The 80th percentile score.

b) If the pass mark was set at 45 marks, estimate from your ogive the number of students who passed the exam.

c) Explain what the interquartile range tells you about the spread of the students' scores.

Item 18

Two market vendors, Aisha and Ben, operating in Owino Market, Kampala, recorded their daily sales (in thousands of UGX) over a period of 30 days. The data is summarized below:

Aisha: Mean Sale = 150, Standard Deviation = 25

Ben: Mean Sale = 160, Standard Deviation = 40

Tasks:

a) Determine which vendor has higher average daily sales.

b) Calculate the coefficient of variation for both Aisha and Ben.

c) Using the coefficient of variation, determine whose sales are relatively more consistent. Justify your answer.

TOPIC 7: SCATTER DIAGRAMS AND CORRELATIONS

Item 19

An agricultural officer in the Bugisu region collects data on the annual rainfall (in mm) and the coffee yield (in kg per tree) for 8 different farms over the past year.

Rainfall (mm), x	Yield (kg/tree), y
1200	2.5
1400	3.0
1000	2.0
1600	3.2
1800	3.5
1100	2.2
1500	3.1
1300	2.8

Tasks:

a) Construct a scatter diagram to visually represent the relationship between rainfall and coffee yield.

b) Based on the scatter diagram, describe the type of correlation you observe between rainfall and yield.

c) By ranking the data for both rainfall (x) and yield (y), calculate Spearman's rank correlation coefficient.

d) Interpret the value of Spearman's rank correlation coefficient you calculated in the context of rainfall and coffee yield in this region. Does it support your observation from the scatter diagram?

Item 20

A teacher at Exodus College School wants to investigate if there's a relationship between the average number of hours students spend studying per week and their score on a recent Physics test. Data for 7 students is collected:

Study Hours/Week (x)	Test Score (y)
5	65
8	75
2	50
10	85
4	60
12	90
6	72

Tasks:

a) Help the teacher to represent this data on a scatter diagram.

b) Visually, draw a line of best fit through the points on your scatter diagram.

c) Comment on the apparent relationship between study hours and test scores based on your diagram and line of best fit.

d) Would it be reasonable to use this relationship to predict the score of a student who studies for 20 hours a week? Explain your reasoning, considering the limitations of extrapolation.

Item 21

An economics student is researching the relationship between the average weekly price of a bunch of Matooke (in UGX) in Nakasero market and the estimated quantity demanded (in hundreds of bunches). Data over 6 weeks is collected:

Price (UGX), P	Quantity (hundreds), Q
5000	80
6000	70
4500	90
7000	60
5500	75

Price (UGX), P	Quantity (hundreds), Q
6500	68

Tasks:

a) Calculate the Spearman's rank correlation coefficient between the price and quantity demanded.

b) Interpret the calculated correlation coefficient. Does it align with typical economic principles of demand?

c) Plot a scatter diagram for the Price (P) vs Quantity (Q). Does the visual pattern support the calculated correlation?

TOPIC 8: DYNAMICS 1

Item 22

Two farm workers, Okello and Lanyero, are pulling a heavy sack of maize (mass 80 kg) across level ground in a Kireka warehouse. Okello pulls with a force of 300 N at an angle of $20°$ above the horizontal. Lanyero pulls with a force of 250 N at an angle of $15°$ above the horizontal, in the same direction as Okello. The coefficient of kinetic friction between the sack and the ground is 0.3. (Assume $g = 9.8\ m/s^2$).

Tasks:

a) Represent all the forces acting on the sack on a diagram.

b) Resolve the forces applied by Okello and Lanyero into horizontal and vertical components.

c) Calculate the total upward vertical component from the workers' pulls and hence determine the Normal Reaction force exerted by the ground on the sack.

d) Calculate the maximum possible frictional force and the total horizontal component of the pulling forces hence determine the net horizontal force acting on the sack.

Item 23

In a mechanics lab, a block A of mass 5 kg rests on a rough inclined plane angled at $30°$ to the horizontal. The coefficient of kinetic friction between block A and the plane is 0.2. Block A is connected by a light inextensible string passing over a smooth pulley at the top of the incline to a block B of mass 3 kg, which hangs freely. The system is released from rest. (Assume $g = 9.8 m/s^2$).

Tasks:

a) Illustrate the forces acting on block A and block B on separate diagrams.

b) For block A, resolve its weight into components parallel and perpendicular to the inclined plane. Calculate the normal reaction force on block A.

c) Determine the frictional force acting on block A as it slides (assume it slides up the plane initially, if unsure, calculate net force without friction first to determine direction).

d) Apply Newton's Second Law to both block A and block B to formulate two simultaneous equations involving the acceleration (a) of the system and the tension (T) in the string. Solve these equations to find the values of a and T.

Item 24

A lorry of mass 5000 kg is parked on a road in Kisoro inclined at an angle θ to the horizontal, where $\sin\theta = 0.1$. The coefficient of static friction between the lorry's tyres and the road is 0.4. The driver has applied the handbrake. We want to determine if the lorry will remain stationary. (Assume $g = 9.8\, m/s^2$).

Tasks:

a) Come up with a diagram showing the forces acting on the lorry assuming it is about to slide down the slope.

b) Resolve the weight of the lorry into components parallel and perpendicular to the road surface, hence find the Normal Reaction force acting on it.

c) Determine the maximum possible static frictional force that can be exerted by the road on the tyres ($F_{\max} = \mu_s N$). Compare this maximum friction with the component of the lorry's weight acting down the slope. Establish wether lorry will remain stationary or slide down. Justify your conclusion.

TOPIC 9: PROBABILITY THEORY

Item 25

In a certain region of Uganda, it is estimated that 2% of the population has a particular disease. A medical test is developed to detect the disease. The test is not perfect:

If a person has the disease, the test correctly gives a positive result 95% of the time (Sensitivity).

If a person does not have the disease, the test correctly gives a negative result 90% of the time (Specificity). A person from the region is selected at random and tested.

Tasks:

a) Construct a tree diagram and use it to calculate the overall probability that a randomly selected person tests positive.
b) Using Bayes' Theorem, determine the probability that a person actually has the disease given that they tested positive.
c) Interpret your result from b) above. What does this tell you about the reliability of a positive test result in this scenario?

Item 26

A factory in Jinja produces light bulbs using three machines: Machine A, Machine B, and Machine C.

Machine A produces 40% of the total output, and 5% of its bulbs are defective.

Machine B produces 35% of the total output, and 3% of its bulbs are defective.

Machine C produces 25% of the total output, and 2% of its bulbs are defective. A bulb is selected at random from the factory's output.

Tasks:

a) Determine the probability that the selected bulb was produced by Machine A AND is defective. Similarly, calculate the probabilities for Machine B being defective and Machine C being defective.
b) Using the results from a), determine the overall probability that a randomly selected bulb from the factory's output is defective.
c) Given that the selected bulb is found to be defective, calculate the probability that it was produced by Machine B.

Item 27

In a class of 60 students at a Kampala school, 40 own an Android phone (A), 25 own an iPhone (I), and 15 own both types.

Tasks:

a) Represent this information on a Venn diagram.

b) Determine the number of students who own:

  i) Only an Android phone.

  ii) Only an iPhone.

  iii) Neither type of phone.

c) A student is selected at random from the class. Calculate the probability that the student owns:

  i) An Android phone or an iPhone.

  ii) Exactly one type of phone.

d) Given that a selected student owns an Android phone, calculate the probability that they also own an iPhone.

TOPIC 10: DIFFERENTIATION 1

Item 28

A farmer in Mukono wants to create a rectangular enclosure for chickens next to a long, straight existing wall. He has 100 meters of fencing wire available for the other three sides of the rectangle. He wants to maximize the area enclosed for his chickens. Let the side parallel to the wall have length x meters, and the other two sides perpendicular to the wall have length y meters each.

Tasks:

a) Help the farmer to express the total length of the fencing used in terms of x and y, and formulate an equation based on the available wire.

b) Express the area A of the enclosure ($A = xy$) as a function of only one variable x. Hence, find the value of x that maximizes the area.

c) Determine the maximum possible area of the enclosure and confirm it is a maximum.

Item 29

The displacement s (in meters) of a particle moving along a straight line from a fixed point O, at time t (in seconds), is given by the equation $s(t) = t^3 - 6t^2 + 9t + 5$, for $t \geq 0$.

Tasks:

a) Determine the expressions for the velocity $v(t)$ and acceleration $a(t)$ of the particle at time t by differentiating the displacement function.

b) Calculate the initial velocity and initial acceleration of the particle $at\ t = 0$.

c) Find the time(s) when the particle is momentarily at rest $v(t) = 0$.

d) Determine the acceleration of the particle at the time(s) when it is at rest. Describe the motion of the particle during the first 4 seconds.

Item 30

A scientist has a spherical balloon is which being inflated. Its radius r is increasing at a constant rate of 0.1 cm per second. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. He wants to find the rate at which the volume is increasing when the radius is 5 cm. He also wants to estimate the approximate increase in volume as the radius increases from 5 cm to 5.1 cm.

Tasks:

a) Help the scientist to determine the rate at which the volume of the bowl is changing with respect to the radius.

b) Determine the rate at which the volume is increasing when the radius $r = 5\ cm$.

c) Estimate the approximate increase in volume $(\delta\ V)$ as the radius increases from $r = 5\ cm$ to $r = 5.1\ cm$.

TOPIC 11: INTEGRATION 1

Item 31

Water flows into a storage tank in Mbale at a rate given by $R(t) = 10 + 0.5t$ liters per minute, where t is the time in minutes from the start $(t = 0)$. The tank was initially empty.

Tasks:

a) Obtain expression for the volume V(t) of water in the tank as an indefinite integral.

b) Using the initial condition, determine the value of the constant of integration C.

c) Calculate the volume of water in the tank after 60 minutes

d) Determine the average rate of flow into the tank during the first 60 minutes using the mean value function.

Item 32

A piece of land is bounded by a river whose shape can be modelled by the curve $y = \sqrt{x}$, the straight line $x = 9$, and the x-axis (representing a straight fence). The coordinates are measured in meters. The owner wants to calculate the area of this piece of land.

Tasks:

a) Help the owner to sketch the region bounded by the land.

b) Set up the definite integral that represents the area of this region hence use it to calculate the exact area of the piece of land.

c) If this area were revolved around the x-axis, it would form a solid shape. Set up the definite integral representing the volume of this solid of revolution hence Calculate this volume.

Item 33

A boda-boda rider accelerates away from a traffic light in Fort Portal. His velocity v (in m/s) after time t (in seconds) is given by $v(t) = 6t - t^2$ for $0 \leq t \leq 6$.

Tasks:

a) Obtain an expression for his displacement $s(t)$ (assume $s(t = 0) = 0$).

b) Determine the displacement of the boda-boda from the traffic light after 3 seconds.

c) Calculate the total distance travelled by the boda-boda in the first 6 seconds.

d) Determine the time t at which the boda-boda reaches its maximum velocity within the interval $0 \leq t \leq 6$. Hence Calculate its maximum velocity.

TOPIC 12: PERMUTATIONS AND COMBINATIONS

Item 34

A student at Ntare School has 4 distinct Mathematics books, 3 distinct Physics books, and 2 distinct Chemistry books. He wants to arrange them on a single shelf.

Tasks:

Help the student to know

a) how many different ways he can arrange 9 books be on the shelf if there are no restrictions?

b) how many ways he can arrange the books if all the Mathematics books must be kept together, all the Physics books must be kept together, and all the Chemistry books must be kept together?

c) In how many ways he can arrange the books if only the Mathematics books must be kept together?

Item 35

Exodus College School needs to form a student committee of 5 members. There are 8 eligible students from Senior Five and 6 eligible students from Senior Six.

Tasks:

a) In how many ways can the committee of 5 be formed if there are no restrictions on the class level?

b) In how many ways can the committee be formed if it must consist of exactly 3 students from Senior Five and 2 students from Senior Six?

c) In how many ways can the committee be formed if it must include at least 4 students from Senior Five?

d) Suppose two specific Senior Six students, Mary and Jane, refuse to be on the committee together. In how many ways can the committee be formed if it must have exactly 3 Senior Five students and 2 Senior Six students, considering this restriction?

Item 36

A mobile banking App requires users to create a 4-digit PIN using the digits 0 to 9.

Tasks:

a) How many different 4-digit PINs can be created if digits can be repeated?

b) How many different 4-digit PINs can be created if digits cannot be repeated?

c) How many different 4-digit PINs can be created if digits cannot be repeated and the PIN must be an even number?

d) How many different 4-digit PINs can be created if digits can be repeated, but the PIN cannot start with 0?