# WUNNA EDUCATIONAL SERVICES

## A-LEVEL MATHEMATICS REVISION QUESTIONS

**SCHOOL NAME:**

___

**STUDENT'S NAME**:

___

**CLASS & STREAM**:

___

***INSTRUCTIONS:***

- Complete all the questions in this Package.
- Submit your work on the first day back after the holiday.
- Ensure all your work is neat and well-organized.
- Make Research but when answering the package ensure that you work independently to ensure that your understanding is reflected.

# A-LEVEL MATHEMATICS TOPICS PER PAPER

## PURE MATHMATICS TOPICS (P425/1)



**1. Analysis (6 questions)**
(a) Differentiation
(b) Integration
(c) Differential equations

**2. Vectors (2 questions)**
(a) Vectors in 2-D
(b) Vectors in 3-D
(c) Ratio theorem
(d) Line and their properties
(e) Planes and their properties

**3. Trigonometry (2 questions)**

**4. Geometry (2 questions)**
(a) Coordinate geometry of lines and triangles
(b) Locus and circles
(c) Parabola

**5. Algebra (4 questions)**
(a) Surds, indices and logarithms
(b) Quadratics
(c) Polynomials
(d) Simultaneous equations
(e) Inequalities
(f) Partial fractions
(g) Complex numbers
(h) Permutation and combinations

# APPLIED MATHEMATICS TOPICS (P425/2)

**1. Mechanics (6 questions)**

(a) Calculus of vectors
(b) General motion of the body
(c) Relative motion
(d) Projectiles
(e) Newtonian mechanics

**2. Numerical analysis (4 questions)**

(a) Location of the roots of anequation
(b) Trapezium rule of numericalintegration
(c) Newton Raphson method
(d) Errors
(e) Flow charts

**3. Statistics and probability(6 questions)**

(a) Mean ,node, median
(b) Index numbers
(c) Correlation coefficient
(d) Scatter diagram
(e) Discrete probability distributions
(f) Continuous probability distributions
(g) Distributions

i. Uniform distribution
ii. Normal distribution
iii. Binomial distribution
iv. Normal approximation to binomial distribution

Estimations

# PURE MATHEMATICS SET ONE (P425/1)



**INSTRUCTIONS:**

- Answer **ALL** the **eight** questions in Section A and any **five** from Section B.
- All necessary working must be shown clearly.

**SECTION A (40 MARKS)**

1. Solve the simultaneous equation:

$p + 2q - r = -1$

$3p - q + 2r = 16$

$2p + 3q + r = 3$ (05mks)

2. Differentiate: $\log_e(1-2x^2)^{\frac{-1}{2}}$ with respect to $x$. (05mks)

3. Find $\int \frac{dx}{1-cosx}$ (05mks)

4. Solve the differential equation $x(1-y)\frac{dy}{dx} + y = 0$ given that

$y = 1 \ when\ x = e$ (05mks)

5. Show that $\frac{sin\theta - 2sin2\theta + sin3\theta}{sin\theta + 2sin2\theta + sin3\theta} = -tan^2\frac{\theta}{2}$ (05mks)

6. Prove by induction that $\sum_{r=1}^{n} 3^{r-1} = \frac{3^n - 1}{2}$ where $n$ is a whole number.(05mks)

7. Solve for x in the equation. $\log_4(6-x) = \log_2 x$ (05mks)

8. Show that $2 + i$ is a root of the equation $2z^3 - 9z^2 + 14z - 5 = 0$.

Hence find the other roots. (05mks)

## SECTION B (60 MARKS)

9. (a) Use Maclaurin's theorem to expand $\frac{1}{\sqrt{1+x}}$ up to the term in $x^3$ (06mks)

(b) Using the binomial theorem expand $(8-24x)^{\frac{2}{3}}$ as far as the 4th term. Hence evaluate $4^{\frac{2}{3}}$ to one decimal place. (06mks)

10. (a) $\int \frac{x^4-x^3+x^2+1}{x^3+x}\,dx$ (07mks)

(b) Evaluate $\int_0^{\pi/2} x\sin^2 2x\,dx$ (05mks)

11. A circle cuts the $y-axis$ at two points $A$ and $B$. It touches the $x-axis$ at a distance 4 units from the origin and distance $AB$ is 6 units. A is a point (0,1): Find the:

(a) Equation of the circle (06mks)

(b) Equations of the tangents to the circle at A and B. (06mks)

12. Sketch the curve $y = \frac{4(x-3)}{x(x+2)}$ (12mks)

13. (a) Determine the coordinates of the point of intersection of the line.
$\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+2}{5}$ and the plane $x+y+z=12$. (06mks)

(c) Find the angle between the line $\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+1}{-1}$ and the plane $x+y+z=12$. (06mks)

14. (a) Given that $x$ and $y$ are real. Find the values of $x$ and $y$ which satisfy the equation.
$\frac{2y+4i}{2x+y} - \frac{y}{x-1} = 0$ (06mks)

(b) Express a complex number $z = 1 - i\sqrt{3}$ in modulus – argument form and hence find $z^2$ and $\frac{1}{z}$ in the form $a+bi$ (06mks)

15. (a) The tenth term of an arithmetic progression (A.P) is 29 and the fifteenth term is 44. Find the value of common difference and the first term. Hence find the sun of the first 60 terms. (07mks)

(b) A cable 10m long is divided into ten pieces whose lengths are in a geometrical progression. The length of the longest piece is 8 times the length of the shortest piece. Calculate to the nearest centimeters the length of the third piece. (05mks)

16.(a)Solve the differential equation: $\frac{dy}{dx} - ytanx = cos^2x$

(b)Given that $y = e^{tanx}$. Show that $\frac{d^2y}{dx^2} - (2tanx + sec^2x)\frac{dy}{dx} = 0$ (12mks)

**END**

# PURE MATHEMATICS SET TWO (P425/1)

## INSTRUCTIONS TO CANDIDATES

- *Answer **all the eight** questions in section **A** and any **five** from section **B**.*
- *Any additional question(s) will **not** be marked.*
- *All working **must** be shown clearly.*
- *Begin each question on a fresh sheet of paper.*
- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

### SECTION A (40 MARKS)

*Answer **all** questions in this section*

1. Solve the equation $\cos(45^0 - x) = 2\sin(30^0 + x)$ for $-180^0 \leq x \leq 180^0$ (05 marks)

2. Solve the inequality (05 marks)

$$\frac{2x^2 - 7x - 4}{3x^2 - 14x + 11} > 2$$

3. Evaluate $\int_0^{\frac{1}{2}\pi} xcosx^2dx$ (05 marks)

4. A circle C, has the equation; $x^2 + y^2 - 2x - 8y - 8 = 0$.

Find the;

(i) Coordinates of its centre (02 marks)

(ii) Shortest distance of the point A$(-5, -4)$ from the circle. (03 marks)

5. A committee of six members is to be chosen from among five men and three women such that atleast two members of each group serve on the committee. Find the number of possible committees that can be formed. (05 marks)

6. Solve the differential equation

$cosec\ x \frac{dy}{dx} = e^x cosecx + 3x$, given that $y\left(\frac{\pi}{2}\right) = 3$. (05 marks)

7. Find the perpendicular distance of the point P (0, 6, 0) from the line with Cartesian equation, $\frac{x+4}{2} = \frac{2-y}{2} = \frac{Z+3}{4}$. (05 marks)

8. Given that: $x = 1 + cos2\theta$ and $y = sin\theta$, show that $\frac{d^2y}{dx^2} = 4\left(\frac{dy}{dx}\right)^3$ (05 marks)

## SECTION B (60 MARKS)

*Answer any **five** question from this section. All questions carry equal marks*

9. (a)Solve the simultaneous equations

$x - 10y + 7z = 13$

$x + 4y - 3z = -3$

$-x + 2y - z = -3$ (05 marks)

(b) When a polynomial $p(x)$ is divided by $x^2 - 5x - 14$, the remainder is $2x + 5$. Find the remainder when $p(x)$ is divided by

(i) $x - 7$

(ii) $x + 2$. (07 marks)

10. (a) Express $4sin\theta - 3cos\theta$ in the form $Rsin\ (\theta - \propto)$; where R is a constant and $\propto$ is an acute angle. Hence solve the equation $4sin\theta - 3cos\theta + 2 = 0$, for $0^0 \leq \theta \leq 360^0$ (07marks)

(b) In any triangle ABC, show that $\frac{a+b-c}{a-b+c} = tan\frac{1}{2}B\ cot\frac{1}{2}C$ (05 marks)

11. The normal to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$ meets the axis of the parabola at G. If GP is produced beyond P to Q such that GP = PQ, show that the equation of the locus of Q is $y^2 = 16a(x + 2a)$. (12 marks)

12. (a) Given the complex numbers $Z_1 = \frac{1+i\sqrt{3}}{2}$ and $Z_2 = \frac{1-i\sqrt{3}}{2}$

(i) Express $Z_1$ and $Z_2$ in polar form

(ii) Find the value of $Z_1^5 + Z_2^5$ (06 marks)

(b) If $-4 - 3i$ is one root of the equation $Z^4 - 4Z^3 - 4Z^2 - 4Z + 925 = 0$, Determine the other roots of the equation. (06 marks)

13. Express $f(x) = \frac{5x^2-8x+1}{2x(x-1)^2}$ into partial fractions. Hence show that

$$\int_4^9 f(x)dx = In\left(\frac{32}{3}\right) - \frac{5}{24}$$ (12 marks)

14. (a)The line $L_1$ passes through the points A and B whose position vectors are $\boldsymbol{3i - i + 2k}$ and $\boldsymbol{-i + j + 9k}$ respectively. Find in vector form, the equation of the line $L_1$. (04 marks)

(b) The line $L_2$ has the equation $\boldsymbol{r} = (8\boldsymbol{i} + \boldsymbol{j} - 6\boldsymbol{k}) + \lambda(\boldsymbol{i} - 2\boldsymbol{j} - 2\boldsymbol{k})$ where $\lambda$ is a scalar parameter.

(i) show that the lines $L_1$ and $L_2$ intersect.

(ii) Determine the position vector of the point of intersection (08 marks)

15. Given the curve; $y = \frac{x^2-x-2}{x^2-x-2}$

(a) Find the:

(i) Equations of the three asymptotes of the curve. (03 marks)

(ii) Stationary point of the curve and determine its nature. (04 marks)

(b) Sketch the curve. (05 marks)

16. (a) Given the curve $\frac{1}{x^2}$, show from the first principles that $\frac{dy}{dx} = \frac{-2}{x^3}$ (06 marks)

(b) If $e^x = \cos(x - y)$, show that $\frac{dy}{dx} = \frac{\sqrt{1-e^{2x}}-e^x}{\sqrt{1-e^{2x}}}$ (06 marks)

# PURE MATHEMATICS SET THREE (P425/1)



**INSTRUCTIONS TO CANDIDATES:**

- *Answer **all** questions in **section A** and any **five** from **section B.***
- *All necessary working must be shown clearly.*
- *Silent non – programmable scientific calculators and mathematical tables may be used.*
- *Any extra question(s) attempted in section **B** will **not** be marked.*

**SECTION A (40 MARKS)**

1. Express $\mathbf{\cos x + 3\sin x}$ in the form $\boldsymbol{R\cos(x-\alpha)}$. Hence solve the equation $\mathbf{\cos 2\theta + 3\sin 2\theta = 2}$ for $0^0 < \theta < 90^0$ *(05 marks)*

2. A line $\mathbf{2x - y + 3 = 0}$ touches a circle whose Centre is **(-4, 5)**. Determine the equation of the circle. *(05 marks)*

3. Solve the following simultaneous equations
$$\mathbf{x + 3y + 2z + 13 = 0}$$
$$\mathbf{2x - 6y + 3z = 32}$$
$$\mathbf{3x - 4y - z = 12}$$
*(05 marks)*

4. In the equation $px^2 + qx + r = 0$, one of the roots is the square of the other. Without solving the equation, show that $q^3 = pr(3q - p - r)$. *(05 marks)*

5. Find $\displaystyle\int \frac{\mathbf{1}}{\mathbf{1+\sin x}}\,\boldsymbol{dx}$ *(05 marks)*

6. By use small changes, show that $\mathbf{\sqrt[5]{244} = 3\frac{1}{405}}$ *(05 marks)*

7. If the position vectors of the points P and Q are $2i + 4j + 6k$ and $-3i + 2j + 8k$ respectively, find the position vector of the point M which divides PQ externally in the ratio 5:3 . *(05 marks)*

8. Find the coefficient of $x^{17}$ in the expansion of $\left(x^3 + \frac{1}{x^4}\right)^{15}$ *(05 marks)*

## SECTION B (60 MARKS)



9. (a) If $\tan X = a$, $\tan Y = b$, $\tan Z = c$. Prove that $\tan(X+Y+Z) = \frac{a+b+c-abc}{1-ab-ac-bc}$.

Hence show that $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{\pi}{4}$. (*06 marks*)

(b) Show that if $\sin(x+\alpha) = k\sin(x-\alpha)$ then $\tan x = \frac{(k+1)}{(k-1)}\tan\alpha$. Hence solve the equation $\sin(x+20^0) = 2\sin(x-20^0)$ for $0^0 \leq x \leq 180^0$. (*06 marks*)

10. (a) Using Maclaurin's theorem, determine the first three non-zero terms of the series for $\log_5(1+e^x)$. (*06 marks*)

(b) Use binomial theorem to obtain the first four terms of the expansion $\sqrt[4]{(1-16x)}$. Hence find $(39)^{\frac{1}{4}}$ correct to 5s.f ( take $x = \frac{1}{10000}$). (*06 marks*)

11. (a) Find the perpendicular distance of a point (3, 0, 1) from the line

$$\frac{x-1}{3} = \frac{y+2}{4} = \frac{z}{12}.$$

(b) Find the Cartesian equation of the plane through points A(2,-1,2), B(0,3,-4) and C(7,4,-1). (**07 marks**)

12. Express $\frac{x+1}{x^2(x^2+1)}$ as partial fractions. Hence evaluate $\int_1^2 \frac{x+1}{x^2(x^2+1)}dx$ (**12 marks**)

13. (a) Given that $Z = \frac{(1-i)(\sqrt{3}-i)}{(1-i\sqrt{3})}$, express Z in polar form. (**04 marks**)

(c) Show that the locus of $\left|\frac{Z-1}{Z+1}\right| = 2$ is a circle. State its centre and radius. (**04 marks**)

(c) Solve the equation $Z^2 - 4(1+i)Z + 9 + 8i = 0$. (**04 marks**)

14. (a) Given that $x$ is a real number, prove that the function $y = \frac{(x+1)(x-3)}{x(x-2)}$ does not lie between 1 and 4.

(b) Determine the turning point(s) and distinguish between them.

(c) State the equations of the asymptotes and the points at which the curve cuts both axes.

(d) Sketch the curve. (**12 marks**)

15. (a) The normal to the parabola $y^2 = 4ax$ at the point A$(at^2, 2at)$ meets the axis of the parabola at T and TA is produced beyond A to B so that $\overrightarrow{TA} = \overrightarrow{AB}$. Show that the equation of the locus of B is $y^2 = 16a(x + 2a)$. **( 06 marks)**

(b) Prove that the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2}=1$ at the point $(a\sec\theta, b\tan\theta)$ is $by + ax\sin\theta = (a^2 + b^2)\tan\theta$. If the normal meets the $x$-axis at P and the y-axis at Q, find the locus of the mid-point of PQ. **(06 marks)**

16. (a) Solve the differential equation $(x + y)\frac{dy}{dx} = x - y, y(3) = -2$. **(05 marks)**

(b) The rate at which a disease spreads through a certain community is found to be directly proportional the fraction $x$ of the community infected after t months but inversely proportional to the fraction not yet infected.

(i) Form a differential equation connecting $x$ and $t$.

(ii) Show that the general solution to the equation can be expressed as $e^{kt} = Axe^{-x}$, where $k$ and $A$ are constants. When first noticed, one half of the community was infected and by this instant the disease is spreading at a fraction $\frac{1}{4}$ per month, show that the particular solution to the differential equation is $e^{t} = 16x^4e^{2-4x}$ **(07 marks)**

## PURE MATHEMATICS SET FOUR (P425/1)

<u>**INSTRUCTIONS TO CANDIDATES**</u>

- *Answer **all** the eight questions in section **A** and any **five** from section **B***
- *Any addition question(s) answered will **not** be marked*
- *All necessary working **must** be clearly shown*
- *Begin each answer on a fresh sheet of paper*
- *Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.*

## SECTION A (40 marks)

*(Answer **all** questions in this section)*



1. Solve for $x$ , in the equation $9^{x-1} - 3^{x+2} + 162 = 0.$ ***(5 marks)***

2. The lines $4x - 3y = 5$ and $y = 3$ are tangents to two circles whose centres lie on the line$x = 7$. Find the distance between the centres of the circles. ***(5 marks)***

3. Solve $sec^2(2\theta) - 3tan2\theta + 1 = 0$ , $for \quad 0^0 \leq \theta \leq 180^0$ ***(5 marks)***

4. The ages of a mother and her three children are in a geometrical progression, the sum of their ages is 195 years and the sum of the ages of the two young children is 60 years. Find the age of the mother. ***(5 marks)***

5. Evaluate $\int_3^5 \frac{2(x+1)}{2x^2-3x+1}\, dx.$ ***(5 marks)***

6. The equation of the normal to the curve $xy^2 + 3y^2 - x^3 + 5y - 2 = 0$ at the point $(a, -2)$ $is$ $15x - 8y - 46 = 0$. Find the value of $a$. ***(5 marks)***

7. Find $\frac{dy}{dx}$ if $y = xsin^2x$ .when $x = \frac{\pi}{4}$ ***(5 marks)***

8. Find the Cartesian equation of a plane containing point (1, 3, - 4) and the line $\frac{x-1}{2} = \frac{y+2}{3} = z.$ ***(5 marks)***

## SECTION B (60 marks)

(Answer any **five** questions from this section. All questions carry equal marks)

9. (a.) Given that $2A + B = 135$ show that $tanB = \frac{tan^2A-2tanA-1}{1-2tanA-tan^2A}$ . ***(4 marks)***

(b.) If $\alpha$ is an acute angle and $tan\alpha = \frac{4}{3}$, show that

$4\sin(\theta+\alpha) + 3\cos(\theta+\alpha) = 5cos\theta$. Hence solve for $\theta$ the equation

$4\sin(\theta+\alpha) + 3\cos(\theta+\alpha) = \frac{\sqrt{300}}{4}$ for $-180^0 \leq \theta \leq 180^0$. *(8 marks)*

10. (a.) Show that $y = x - 3$ is a tangent to the curve $y = x^2 - 5x + 6$. *(3 marks)*

(b.) A chord to the parabola $4x - 3y^2 = 0$ is parallel to the line $2x - y = 4$ and passes through point (1, 1). Find;

(i.) the equation of the chord.

(ii.) The coordinates of the points of intersection of the chord with the parabola.

(iii.) The acute angle between the chord and the directrix of the parabola. *(9 marks)*

11. (a.) Expand $(4-3x)^{\frac{1}{2}}$ in ascending powers of x up to the term in $x^3$. Taking $x = \frac{1}{25}$ find $\sqrt{97}$ . *(8 marks)*

(b.) Find the term independent of x in the binomial expansion of $\left(2x - \frac{1}{x^2}\right)^9$.

*(4 marks)*

12. (a.) Solve for x and y values in the equation; $\frac{x}{2+3i} + \frac{y}{3-i} = \frac{6-13i}{9+7i}$. *(6 marks)*

(b.) Given that $-4 + i$ is a root of the equation $z^4 + 6z^3 + 6z^2 + 6z + 65 = 0$, find the other roots of the equation and represent the roots in polar form. *(6 marks)*

13. (a.) Find the volume of a solid generated by rotating about the y-axis, the area enclosed by the curve $y^2 + 4x = 9$ , the y-axis and $y = -2$. *(5 marks)*

(b.) Find $\int xln(2x)dx$ . *(3 marks)*

(c.) Evaluate $\int_0^1 \frac{2x-1}{(x-3)^2}\,dx$ . *(4 marks)*

14. The points A, B, C and D are given by the coordinates (5, 2, -3), (-1, 0, -1), (9, 5, -8) and (5, 7, -14) respectively. If lines AB and CD intersect at point E. Find;

(i.) Equations of lines AB and CD.

(ii.) Coordinates of point E

(iii.) The acute angle between lines AB and CD. ***(12 marks)***

15. A curve is given by the parametric equations; $x = 3t$ and $x = \frac{2t^2}{1-t}$.

(a.) Find the Cartesian equation of the curve.

(b.) Sketch the curve, showing clearly the asymptotes and turning points. ***(12 marks)***

16. (a.) Solve the differential equation $\frac{dy}{dx} = 4x - 7$, given that $y(2) = 3$. ***(3 marks)***

(b.) The rate at which a candidate was losing support during an election campaign was directly proportional to the number of supporters he had at that time. Initially he had $V_0$ supporters and t weeks later, he had V supporters.

(i.) Form a differential equation connecting V and t.

(ii.) Given that the supporters reduced to two thirds of the initial number in 6 weeks, solve the equation in (i.) above.

(iii.) Find how long it will take for the candidate to remain with 20% of the initial supporters. ***(9 marks)***

**END**

# PURE MATHEMATICS SET FIVE (P425/1)

**INSTRUCTIONS TO CANDIDATES:**

- Attempt **all** the **eight** questions in **Section A** and**Not** more than **five** from **Section B.**
- Any additional question(s) will not be marked.
- All working must be shown clearly.
- Silent non-programmable calculators and mathematical tables with a list of formulae may be used.
- Graph papers are provided.

## SECTION A: (40MARKS)

*Answer **all** the **eight** questions in this Section.*

1. Solve the simultaneous equations; $\frac{1}{2y} + \frac{1}{x} = 4$ ; $\frac{3}{x} - \frac{1}{y} = 7$. (5marks)

2. Prove that; $\frac{log_2 x - log_2 x^2}{log_4 x^3} + \frac{5}{3} = log10$. (5marks)

3. Given the parabola $y^2 = 8x$,

a) Express a point $T$ on the parabola in parametric form using $t$ as the parameter. (2marks)

b) If parameter $r$ gives point $R$, show that the gradient of chord $TR$ is $\frac{2}{t+r}$.(3marks)

4. Find $\int x^3 e^{x^2} dx$. (5marks)

5. The line $r = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ a \\ b \end{pmatrix}$ meets a plane $P$ perpendicularly at the point $(3, 1, 2)$.

Find the vector equation of the plane. (5marks)

6. Solve $\sin(120^0 + 3x) = cos(90^0 - x)\ for\ 0^0 \leq x \leq 90^0$. (5marks)

7. A roll of fencing material $152m$ long is used to enclose a rectangular area using two existing perpendicular walls. Find the maximum area enclosed. (5marks)

8. Solve the differential equation $\frac{dy}{dx}x - x = y$ given that $y = e$ when $x = e$. (5marks)

## SECTION B : (60MARKS)

9. a) Prove that; ${}^{n+1}_{r+1}C + {}^{n+1}_{r+2}C = {}^{n+2}_{n-r}C$. (6marks)

b) Two blue, three red and four black beads are to be arranged on a circular ring made of a wire so that the red are separated. Find the number of different arrangements. (6marks)

10. Given that; $f(x) = \frac{1+2x}{1-x}$

a) Find Maclaurin's expansion of $f(x)$ upto the term in $x^3$. (8marks)

b) Hence, find the value of $\frac{1.02}{0.99}$ to four significant figures. (4marks)

11. a) Given that; $ysinx + xcosy = \frac{\pi}{2}$, find $\frac{dy}{dx}$. (4marks)

b) A square prism is always three times the width in length. If the volume increases at a constant rate of $4cm^3s^{-1}$, find the rate of change of the cross-sectional area when the width is $12cm$. (8marks)

12.

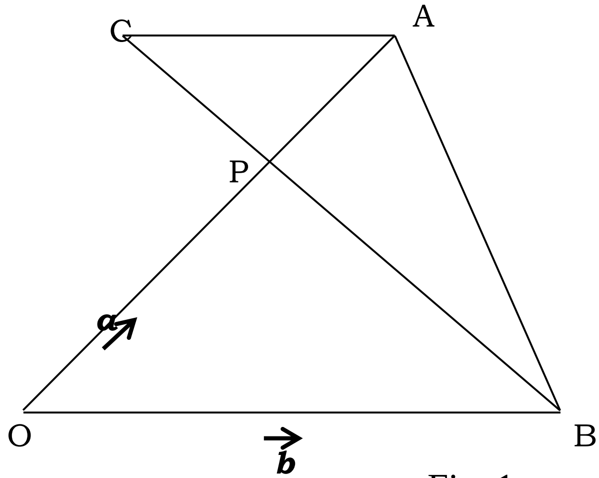



Fig. 1

Figure 1 shows points A and B with position vectors $a$ and $b$ respectively. $3AC = BO$.

a) Express each of the following in terms of vectors $a$ and $b$.

i) $BA$ (2marks)

ii) $BC$ (3marks)

b) Find the ration $BP : PC$ (7marks)

13. a) Prove that $cos(tan^{-1}x) = (x^2+1)^{-\frac{1}{2}}$. (4marks)

b) i) Prove that $\frac{cos^2 4x + cos4x + sin^2 4x}{cos^2 4x - cos4x + sin^2 4x} = 3$ for $0 \leq x \leq \pi$. (4marks)

14. The lines $L_1$ and $L_2$ are perpendicular and intersect at $P(0,5)$. Line $L_1$ meets the x-axis in the first quadrant at $Q$ such that $PQ = 13$ units. If $L_2$ meets the x-axis at $R$, without graphical construction, find the area of the triangle $PQR$. (12marks)

15. Given that $Z1 = 2 - 3i, Z_2 = 1 + 2i$ $and$ $Z_3 = 3 - 4i$.

a) Express $\frac{Z_1+Z_2}{Z_1Z_2}$ in the form $a + bi$ where $a$ and $b$ are real numbers.(6marks)

b) Find a polynomial $p(x)$ of degree four where the roots of $p(x) = 0$ are $Z_2$ and $Z_3$. (6marks)

16. Evaluate; $\int_2^3 \frac{x^4-x^3-x^2+4x-1}{(x-1)(x^2+1)} dx$. (12marks)

**END**

# PURE MATHEMATICS SET SIX (P425/1)

**INSTRUCTIONS TO CANDIDATES:**

- Answer **all** the eight questions in section **A** and any **five** questions from section **B**.
- Any additional question(s) answered will **not** be marked.
- Show **all** necessary working clearly.
- Begin each answer on a fresh page of paper.
- Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

## SECTION A (40 MARKS)

Answer **all** questions in this section.

1. Solve the simultaneous equations.
$p + q + r = 0$, $p + 2q + 2r = 2$ and $2p + 3r = 4$. (05marks)

2. Determine the Cartesian's equation of a line passing through points A (2,5,4) and B (5,3,7) (05marks)

3. A Circle with Centre C, cuts another circle $x^2 + y^2 - 4x + 6y - 7 = 0$ at right angles and passes through the point (1, 3). Find the locus of Centre C. (05marks)

4. Solve the equation $\tan^{-1}(2x+1) = \tan^{-1}(2) - \tan^{-1}(2x-1)$ (05marks)

5. Evaluate $\int_0^{\sqrt{\frac{\pi}{2}}} \frac{x}{1+\sin(x^2)} dx$ (06marks)

6. A committee of four pupils is to be selected from three boys and seven girls. How many committees are formed in order to have girls as the majority in committee? (04marks)

7. Use Maclaurin's theorem to expand $In\sqrt{\left(\frac{1+2x}{1+x}\right)}$ up to $x^2$ (05marks)

8. The inside of a glass is in the shape of an inverted cone of depth 8cm and radius 4cm full of wine. The wine is leaking from small hole at vertex at rate 0.06cm$^3$s$^{-1}$ into somebody mouth. Find the rate at which surface area of wine in contact with glass in decreasing when depth is 6cm. (05marks)

## SECTION B (60 marks)

Answer any **five** questions from this section.

9. (a) Solve inequality; $\frac{x-2}{x+1} \geq \frac{x+1}{x+3}$ (06marks)

(b) John deposits Shs. 3,000,000 at beginning of every year in a micro-finance bank starting 2015, how much would he collect at the end of 2020 if the bank offers compound interest of 12.5% per annum and no withdrawal is made within the period. (06marks)

10. (a) Find the vector equation of the line passing through the point (3,1, 2) and perpendicular to the plane r.$(2i - j + k) = 4$. Hence find point of intersection of line and the plane. (06marks)

(b) The position vectors of the points A, B and C are $2i - j + 5k, i - 2j + k$ and $3i + j - 2k$ respectively. Given that L and M are mid-points of AC and CB respectively. Show that BA = 2ML (06marks)

11. (a) Solve Cos3θ + Cos2θ + Cosθ = 0, $0^0 \leq \theta \leq 180^0$ (05marks)

(b) Show that sin3θ =3sinθ - 4sin$^3$θ, hence find all solutions of the equation $8x^3 - 6x + 1 = 0$. Correct to 3 decimal places. (07marks)

12. Given curve $y = \frac{(x-3)^2}{(x-9)(x-1)}$. Find equations of asymptoles and sketch the curve. (12marks)

13. Express $f(x) = \frac{x^3 + 4x^2 - 5x - 4}{(x-2)^2(1+x^2)}$ into partial fractions, hence evaluate $\int_3^5 f(x)dx$. (12marks)

14. (a) Solve $x\frac{dy}{dx}+2y=x^2$ when y (1) = 1. (05marks)

(b) A liquid cools in the environment of a constant temperature of $21^0$C at the rate proportional to the excess temperature. Initially the temperature of liquid is $100^0$C and after 10 minutes the temperature dropped by $16^0$C. Find how long it takes for the temperature of liquid to be $70^0$C. (07marks)

15. (a)Given that the root of $z^4-4z^3+3z^2+3z^2 2z-6=0$ is 1-i, find other roots. (06marks)

(b) Evaluate $\left(1+i\sqrt{3}\right)^{\frac{2}{3}}$ (06marks)

16. (a) Find the equation of a circle which is a tangent to the lines $3y=4x$, $y=8$ and $4x+3y=0$ (05marks)

(b) If the line $y=mx+c$ is a tangent to the ellipse $a^2y^2+b^2x^2=a^2b^2$, prove that $C^2 = b^2+a^2m^2$. Hence determine the equations of the common tangents to ellipse $4x^2+14y=56$ and $3x^2+23y^2=69$ (07marks)

# PURE MATHEMATICS SET SEVEN (P425/1)

## INSTRUCTIONS TO CANDIDATES

- Answer **all** the eight questions in section **A** and any **five** from section **B**
- Any addition question(s) answered will **not** be marked
- All necessary working **must** be clearly shown
- Begin each answer on a fresh sheet of paper
- Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

### SECTION A (40 MARKS).

**Attempt all the questions in this section**

1. If $\alpha^2$and $\beta^2$ are the roots of $x^2-21x+4=0$ and that $\alpha$ and $\beta$ are both positive, find an equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ ( 5marks)

2. A cylinder has radius $r$ and height $h$. The sum of $r$ and $h$ is 2. Find the maximum volume of the cylinder in terms of $\pi$ (5marks)

3. Evaluate $\int_{In2}^{In5} \frac{2}{e^{x}-e^{-x}}dx$ (5marks)

4. The first term of a geometric progression is $18$ and the sum to infinity is $20$ .Find the common ratio and the sum of the first $6$ terms. (5marks)

5. Find the area bounded by the curve $y=5x-x^{2}$ and the line $y=x$ (5marks).

6. Determine the angle between the line $r=\begin{pmatrix}1\\2\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and the plane $2x-y+z=4$

7. Prove that $(\cos ecA-\sin A)(\sec A-\cos A)=\frac{1}{\tan A+\cot A}$ (5marks)

8. The line $y=3x-4$ is a tangent to the circle whose centre is the point $(5,2)$.find the radius of the circle (5marks)

**SECTION B (60marks)**

Attempt any **5 questions** from this section. All questions carry equal marks.

9. Given the lines $l_1$ and $l_2$ are $l_1: r=\begin{pmatrix}1\\-1\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ and $l_2; \begin{pmatrix}1\\2\\2\end{pmatrix}+\mu\begin{pmatrix}-3\\0\\4\end{pmatrix}$ intersect

a). Find the point of intersection of the lines (6marks)

b). Find a vector equation of the plane containing the line in (a) above (6marks)

10a).Solve the equation $2\cos\theta\cos 2\theta+\sin 2\theta=2(\cos^{3}\theta-\cos\theta)$ for $0^{0}\le\theta\le 360^{0}$ (7marks)

b). Prove that $\tan^{-1}x+\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{\pi}{4}$ (5marks)

11a). The complex number $z$ satisfies the equation $2z\overset{*}{z}-4z=3-6i$ ,where $\overset{*}{z}$ is a complex Conjugate of $z$. Find the possible values of $z$ in the form $x+iy$

(6marks)

b) Use Demoivres theorem to find the four roots of the equation $z^4-\sqrt{3}+i=0$

(6marks)

12a). Given that $\log_9 xy=6$, prove that $\log_3 x+\log_3 y=12$. Hence solve the simultaneous equations $\log_9 xy=6$ and $(\log_3 x)(\log_3 y)=20$ (7marks)

b). If $y=\dfrac{x^2+3}{x-1}$ , where $x$ is real , show that $y$ cannot take any value between $-2$ and $6$ (5marks)

13 a). The surface area of a cube is increasing at a rate of $10cms^{-1}$. Find the rate of increase of the Volume of the cube when the edge is of length $12cm$ (6 marks)

b). Prove that $\int_0^{\frac{\pi}{2}} x^2\sin x\cos x dx=\dfrac{\pi^2}{16}-\dfrac{1}{4}$ (6marks)

14a).Use the substitution $t=\tan x$ to find the integral $\int\dfrac{1}{\cos 2x-3\sin^2 x}dx$ (6marks)

b). Show that the integral $\int\dfrac{x}{2x^2-x+1}dx=\dfrac{1}{4}\log_e(2x^2-x+1)+\dfrac{1}{2\sqrt{7}}\tan^{-1}\left(\dfrac{4x-1}{\sqrt{7}}\right)+c$

(06 marks)

15a). Prove that the line $5y-4x=25$ touches the curve $9x^2+5y^2=225$ (5marks)

b). Show that the equation of the tangent to the curve $bx^2+ay^2=a^2b^2$ at the point with parametric equations $x=a\cos\theta, y=b\sin\theta$ is $\dfrac{x}{a}\cos\theta+\dfrac{y}{b}\sin\theta=1$.

This tangent meets the $x-$axis at $A$ and $y-$axis at $B$. Find the area of the triangle $OAB$ (7marks)

16. A hot body of temperature of $80^0C$ is placed in a room of temperature $22^0C$, $12^0C$ minutes later its temperature is $72^0C$. Form a differential equation to represent the rate of change of temperature, $\theta$ of the body with time, $t$ (9marks).

ii) Determine the temperature of the body after $30^0C$ minutes (3 marks)

**END**

# PURE MATHEMATICS SET EIGHT (P425/1)

**INSTRUCTIONS**

- Answer all the questions in section **A** and only **FIVE** questions in section **B**
- Show all necessary working clearly
- Silent non-programmable Scientifics calculators and mathematical tables with a list of formula may be used.

**SECTION A (40 MARKS)**

Answer all questions in this section.

1. Solve the simultaneous equations

$$8^{x-y} = 4^{x+y}, 5^{x^2-y^2} = 15625$$ **(05 marks)**

2. The second term of an arithmetic progression is -4 and the sixth term is -24. Find the fifteenth term and the sum of the first fifteen terms of the progression. **(05 marks)**

3. Find the volume generated when the area bounded by the curve $y = 5cos2x$, the x-axis and the ordinates $x = 0$ and $x = \frac{\pi}{4}$ is rotated about the x-axis through a complete rotation. **(05marks)**

4. Differentiate with respect to $x$ ; $log_{10} \frac{e^x}{\text{Cos } 3x}$ . **(05 marks)**

5. If $\sin 2\theta = \cos 3\theta$ find values of $\sin\theta$, hence solve the equation $\sin 2\theta - \cos 3\theta = 0$ for $0^0 \leq \theta \leq 360^0$ . **(05 marks)**

6. Evaluate $\int_1^2 \frac{dx}{x\sqrt{x^2-1}}$ by use of the substitution; $x = \frac{1}{u}$ **(05 marks)**

7. Solve the differential equation $\frac{dR}{dt} = e^{2t} + t$, given that $(0) = 3$. **(05 marks)**

8. Find the acute angle between the line $\frac{x-4}{2} = \frac{y+1}{-1} = \frac{1-z}{2}$ and the plane $6x + 2y - z = -4$. **(05 marks)**

## SECTION B (60 MARKS)

*Answer any **FIVE** questions from this section. All questions carry equal marks.*

9. (a) Given that the vectors **i** – p**j** + **k** and -3**j** + 2**j** + 4**k** are perpendicular, determine the value of p. **(02 marks)**

(b) Find the angle between the lines;

$r_1 = (1+\lambda)\boldsymbol{i} + (1-\lambda)\boldsymbol{j} + (2+\lambda)\boldsymbol{k}$ and $r_2 = (1-\mu)\boldsymbol{i} + (1-2\mu)\boldsymbol{j} + (1+\mu)\boldsymbol{k}$ **(05 marks)**

(c) Show that the line $x + 1 = y = \frac{z-3}{2}$ is parallel to the plane $r.(\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}) = 3$ and find the distance between them. **(05 marks)**

10. (a) If $z = x + iy$, show that $arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ is a circle. Find its centre and its radius. **(06 marks)**

(b) Z and $\bar{Z}$ are conjugate complex numbers. Find the values of Z that satisfies the equation; $3Z\bar{Z} + 2(Z - \bar{Z}) = 39 + 12i$ **(06 marks)**

11. If y $= \frac{2x^2 + 14x\ \ 10}{2x^2 + 9x + 4}$, express y in partial fractions. Hence determine $\int y dx$. **(12 marks)**

12. (a) If $y = \frac{x\ (1+x^2)^{3/2}}{\sqrt{1-x^2}}$, find $\frac{dy}{dx}$ in its simplest form. **(06 marks)**

(b) Use Maclaurin's theorem to expand $\tan^{-1} x$ by taking the first three **non-zero** terms. Hence, evaluate $\mathbf{tan^{-1}\, 0.1}$ **,** give your answer to 4 decimal places. **(06 marks)**

13. (a) Solve the equation ;

$tan^{-1}(2x+1) + tan^{-1}(2x-1) = tan^{-1}(2)$ **(06 marks)**

(b) Solve the equation $tan^2 x - sin^2 x = 1;\ for\ 0 \le x \le 2\pi$. **(06 marks)**

14. (a) Obtain the expansion in a ascending powers of x of $(1+2x)^{15}$ as far as the term in $x^3$. Hence evaluate $(1.002)^{15}$ correct to 5 decimal places. **(06 marks)**

(b) An amount of shs.2000 is invested at an interest of 5% per month. If shs. 2000 is added at the beginning of each successive month but no withdrawals.

(i) Give an expression for the value accumulated after n months.

(ii) After how many months will the amount have accumulated first exceed shs. 42000? **(06 marks)**

15. Newton's law of cooling states that the rate at which a body cools is directly proportional to the excess temperature of the body over the temperature of its surroundings. Given that at time **t** minutes a body has a temperature **T$^0$C** and its surroundings a constant temperature $\boldsymbol{\theta^0 C}$, form a differential equation in terms of **T,Q, t** and the constant of proportionality $\boldsymbol{K}, \boldsymbol{K} > 0$. Integrate this equation to show that $\mathbf{ln}(\boldsymbol{T} - \boldsymbol{\theta}) = -\boldsymbol{kt} + \boldsymbol{c}$. where **C** is a constant.

At 2:33pm, the water in a kettle boils at 100$^0$C in a room of constant temperature 21$^0$C. After 10 minutes, the temperature of the water in the kettle is 84$^0$C. Use this information to find **c** and **k**, hence find the time taken for the water in the kettle to have the temperature of 70$^0$c **(12 marks)**

16. (a) Find the values of **m** for which the line $\boldsymbol{y} = \boldsymbol{mx}$ is a tangent to the circle $\boldsymbol{x^2 + y^2 + fy + c = 0}$ **(03 marks)**

(b) Find the points where the line $\mathbf{2}\boldsymbol{y} - \boldsymbol{x} + \mathbf{5} = \mathbf{0}$ meets the circle

$\boldsymbol{x^2 + y^2 - 4x + 3y - 5 = 0}$ obtain the equation of the obtain the equation of the tangents and normals to the circle at these points. **(09 marks)**

**END**

# PURE MATHEMATICS SET NINE (P425/1)

**INSTRUCTIONS TO CANDIDATES:**

- *Answer **all** the **eigh**t questions in section A and any **five** from section B.*
- *Additional questions answered will not be marked.*
- *All necessary working must be shown clearly.*
- *Begin each answer on a fresh sheet of paper.*
- *Graph paper is provided.*
- *Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

**SECTION A (40 MARKS)**

*(Answer **all** questions in this section.)*

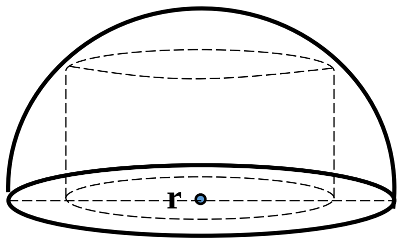
1. From the result $\sum_1^n r^3 = \frac{n^2}{4}(n+1)^2$, deduce that

$(n+1)^3 + (n+2)^3 + \cdots + (4n)^3 = \frac{1}{4}n^2(17n+5)(15n+3)$. (5 marks)

2. Lines $L_1$ and $L_2$ meet along the y-axis and enclose an area of 10.5 square units with the x-axis. If the equation of the line $L_1$ is $3x - 5y + 15 = 0$; find the equation of line $L_2$. (5 marks)

3. Solve $\sin\frac{x}{2} - \cos\frac{3x}{2} = 0$ , for $0 < x < \pi$. (5 marks)

4. A cylindrical tube (open at both ends) is inscribed in a semi- hemisphere of radius $r$ as shown in the figure below.




Find the maximum area of curved surface of the thin material from which the cylindrical tube is derived in terms of $r$. (5 marks)

5. Find the Cartesian equation of the plane containing the points $A(1,1,3)$, $B(3,2,4)$ and $C(4,3,3)$. (5 marks)

6. Solve the simultaneous equations;

$x^2 - 3xy + y^2 = 11, \qquad x - 2y + 5 = 0.$ (5 marks)

7. Evaluate $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{1}{(1+x^2)\tan^{-1} x} dx$ (5 marks)

8. Find the general solution of the differential equation:

$\frac{dy}{dx} = \frac{y^2-1}{2\tan x}$. (5 marks)

## SECTION B (60 MARKS)

*(Attempt any **5** questions from this section. All question carry equal marks)*

9. (a)A polynomial, $P(x)$ of degree 2 leaves remainders of $-1$, 25 and 5 when divided by $(x-1)$, $(x-3)$ and $(x+2)$ respectively. Find$P(x)$. (6 marks)

(b)A newly married couple agreed to invest $Shs$. 4,500,000 at the beginning of every year starting January, 2019 with Jubilee insurance Company. The company pays a compound interest 15% per annum. Determine the total amount of money the couple would have accumulated by the end of December, 2027. (6 marks)

10. (a) A circle has its center as the point $(2,10)$. Point $F(-1,13)$ is the furthest point on the circle from the line $y = x$. Find,

(i) The equation of the circle. (3 marks)

(ii) The shortest distance between the circle and the line $y = x$.( 3marks)

(iii) The coordinates of N the point on the circle nearest to the line $y = x$.

(3 marks)

(b) Find the equation of the circle with centre $(1,-7)$ that is orthogonal to the circle in (a) (i) above. (3 marks

11. (a) Show that in any triangle ABC, if $2s = a + b + c$,

$$1 - \tan\frac{1}{2}A\ \tan\frac{1}{2}B = \frac{c}{s}.$$

(b) Prove that $4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right) = \frac{\pi}{4}$.

12. (a) Find the Cartesian equation of the locus of the point, P corresponding to the values of $z$ for which $Re(z+1) = |z-1|$ and hence, represent this locus on an Argand diagram. (5 marks).

(b) Given that $z = 1 - i$, find real numbers $a$ and $b$ such that

$$\frac{a}{2z-3} + \frac{b}{1-z^3} = -4i.$$ (7 marks).

13. (a) The point A has position vector $\boldsymbol{i} + 4\boldsymbol{j} - 3\boldsymbol{k}$ with reference to the origin, O. The line, L has vector equation $\boldsymbol{r} = t\boldsymbol{j}$. The plane, $P$ contains the line L and the point A. Find the Cartesian equation for the plane, $P$.

(b) Find;

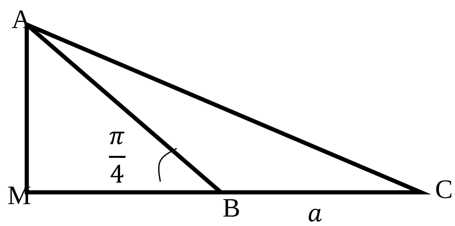
(i) the point of intersection of the plane , $P$ and the line

$$\boldsymbol{r} = 15\boldsymbol{i} - 8\boldsymbol{j} + 3\boldsymbol{k} + \mu(4\boldsymbol{i} + 3\boldsymbol{j}).$$

(ii) $\sin\theta$, where $\theta$ is the acute angle between the plane, $P$ and the line

$$\boldsymbol{r} = 15\boldsymbol{i} - 8\boldsymbol{j} + 3\boldsymbol{k} + \mu(4\boldsymbol{i} + 3\boldsymbol{j}).$$

14 In the triangle ABC, the $angle\ ABC = \frac{3\pi}{4}$ and $BC = a$.




(a) The angle ACB is $\theta$. Show that the length $x$ of the perpendicular $AM$ from $A$ to $CB$ produced is given as $x = \frac{a}{\cot\theta - 1}$. (3 marks)

(b) An increment $\delta\theta$ in the angle ACB produces an increment $\delta x$ in $AM$. Show that $\delta x \approx \frac{x^2}{a} cosec^2\theta\ \delta\theta$. (5 marks)

(c) In case when $\tan\theta = \frac{1}{2}$, show that an error of $1^o$ in the measurement of $\theta$ would result in an error in the deduced value of $x$ of about 9%. (4 marks)

15. Given that $f(x) = \frac{3x^3 - x - 2}{x^2(x^2+x+1)}$, evaluate $\int_2^3 f(x)\ dx$. (12 marks)

16. Ebola disease was found to spread in a certain community at a rate proportional to the fraction $x$ of the community infected at time $t$ months, but inversely proportional to the fraction not yet infected.

(a) Set up a differential equation connecting $x$ and $t$. Show that the general solution to the equation can be expressed as : $e^{kt} = Axe^{-x}$, where $A$ and $k$ are constants. (4 marks)

(b) When first noted, a half of the community was infected and at this instant the disease was spreading at a fraction of $\frac{1}{4}$ per month.

(i) Show that the particular solution to the differential equation can be expressed as: $e^t = 16x^4e^{2-4x}$ . (4 marks)

(ii) Find how long (in days ) it takes for the whole community to be infected from the instant the disease was first noticed. (4 marks)

**END**

# PURE MATHEMATICS SET TEN (P425/1)



***INSTRUCTIONS TO CANDIDATES:***

- *Answer **all** the **eight** questions in Section **A** and only **five** questions in Section **B**.*
- *Indicate the five questions attempted in section B in the table aside.*
- *Additional question(s) answered will **not** be marked.*
- ***All** working **must** be shown clearly.*
- *Graph paper is provided.*
- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

**SECTION A (40 MARKS)**

**Qn 1:** An arithmetic progression contains $\boldsymbol{n}$ terms. The first term is 2 and its common difference is $\frac{2}{3}$. If the sum of the last four terms is 72 more than the sum of the first four terms, find $\boldsymbol{n}$. [5marks]

**Qn 2:** Find the equation of a circle which touches the line $3x + 4y = 9$ has a centre $(4, -7)$ [5marks]

**Qn 3:** Differentiate $\mathbf{cos}\,\boldsymbol{x}$ from first principles. [5marks]

**Qn 4:** Four letters of the word "**HYPERBOLA**" are to be arranged in a row. In how many of these arrangements are the vowels separate? [5marks]

**Qn 5:** Solve for $x$, $2\sin^2\left(\frac{x}{2}\right) - \cos x + 1 = 0$, where $0 \leq x \leq 2\pi$. [5marks]

**Qn 6:** Prove that the integral of cosec $\left(\frac{x}{2}\right)$ for $x$ between $\pi$ and $\frac{4\pi}{3}$ is $\ln 3$. [5marks]

**Qn 7:** Find the shortest distance of a point $A(1, 6, 3)$ from the line

$$\underset{\sim}{r} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k} + \beta\left(-\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}\right).$$ [5marks]

**Qn 8:** The surface area of a sphere is decreasing at a rate of $0.9\ \text{m}^2/\text{s}$ when the radius is $0.6$ m. Find the rate of change of the volume of the sphere at this instant. [5marks]

**SECTION B (60 MARKS)**



**Question 9:**

(a). If the roots of the equation $x^2 + (x+1)^2 = k$ are $\alpha$ and $\beta$;

(i). Prove that $\alpha^3 + \beta^3 = \frac{1}{2}(1 - 3k)$.

(ii). Find a quadratic equation whose roots are $\alpha^3$ and $\beta^3$.

(b). (i). Given that $|x| < \frac{1}{2}$, expand $\frac{1+5x}{\sqrt{1+2x}}$ upto the term in $x^3$.

(ii). By substituting $x = 0.04$ in (b)(i) above, deduce the approximation of $\frac{1}{\sqrt{3}}$ correct to 4 decimal places. [12marks]

**Question 10:**

Given that $y = \frac{\sin x - 2\sin 2x + \sin 3x}{\sin x + 2\sin 2x + \sin 3x}$

(i). Prove that $y + \tan^2\left(\frac{x}{2}\right) = 0$, and hence express the exact value of $\tan^2 15°$ in the form $p + q\sqrt{r}$ where $p$, $q$ and $r$ are integers.

(ii). Hence find the value of $x$ between $0°$ and $360°$ for which

$2y + \sec^2\left(\frac{x}{2}\right) = 0$. [12marks]

**Question 11:**

Given the curve $f(x) = \frac{2x^3 - x^2 - 25x - 12}{x^3 - x^2 - 5x + 5}$ ;

(a). Find the:

(i). value of $x$ for which $f(x) = 0$.

(ii). assymptotes for $f(x)$.

(iii). $x$ and $f(x)$ intercepts for the curve.

(b). Sketch the curve. [12marks]

## Question 12:

A point representing the complex number $Z$ moves such that $\left|\frac{Z-2}{Z-4}\right| > \frac{1}{2}$

(i). Prove that the locus of $Z$ is a circle.

(ii). Find the centre and radius of this circle.

(iii). Represent $Z$ on the argand diagram.

(iv). State the least and greatest values of $|Z|$. [12marks]

## Question 13:

(a). Given two vectors $\underset{\sim}{\boldsymbol{a}} = 3\underset{\sim}{\boldsymbol{i}} - 12\underset{\sim}{\boldsymbol{j}} + 4\underset{\sim}{\boldsymbol{k}}$ and $\underset{\sim}{\boldsymbol{b}} = \underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{k}}$ ; find:

(i). the angle between $\underset{\sim}{\boldsymbol{a}}$ and $\underset{\sim}{\boldsymbol{b}}$ ,

(ii). a vector that makes a right angle with $\underset{\sim}{\boldsymbol{a}}$ and with $\underset{\sim}{\boldsymbol{b}}$ .

(b). Find the equation of the plane passing through the points $\boldsymbol{A}(1,1,0)$, $\boldsymbol{B}(3,-1,1)$, $\boldsymbol{C}(-1,0,3)$ and find the shrotest distance of the point $(3,2,1)$ to the plane. [12marks]

## Question 14:

(a). Using calculus of small increments, or otherwise, find $\sqrt{98}$ correct to one decimal place. [4marks]

(b). Use Maclaurin's theorem to expand $\ln(1+ax)$, where $a$ is a constant. Hence or otherwise expand $\ln\left(\frac{1+x}{\sqrt{1-2x}}\right)$ up to the term in $x^3$. For what value of $x$ is the expansion valid? [8marks]

## Question 15:

A tangent to the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ at a point, $P(6\cos\theta, 4\sin\theta)$ meets the minor axis at $\boldsymbol{A}$. If the normal at $\boldsymbol{P}$ meets the major axis at $\boldsymbol{B}$, find the:

(i). Coordinates of $\boldsymbol{A}$,

(ii). Coordinates of $\boldsymbol{B}$,

(iii). Locus of the midpoint of $\boldsymbol{AB}$. [12marks]

**Question 16:**

(a). Find the general solution of $(x^2+1)\frac{dy}{dx}+2x-2xy=0$

(b). A moth ball evaporates at a rate proportional to its volume, losing half of its volume every 4 weeks. If the volume of the moth ball is initially 15 $cm^3$ and becomes ineffective when its volume reaches 1 $cm^3$, how long is the moth ball effective? [12marks]

***END***

# PURE MATHEMATICS SET ELEVEN (P425/1)

**INSTRUCTIONS:**

- Attempt all questions in Section A and any FIVE in section B.
- Begin every question on a fresh page.
- Show all the necessary working.

**SECTION A**

1. Solve $\log_2 x + \log_x 16 = 4$ (5 marks)

2. Find the Cartesian equation of the locus Z of $|Z-2+i|=1$. (5 marks)

3. Find the Cartesian equation of a line through points (2, 0, 1) and (-1, 4, 1). (5 marks)

4. Solve the equation: $2\cos\alpha + 3\sin\alpha = 5$ for $-\pi \le \alpha \le \pi$. (5 marks)

5. Evaluate $\int_0^1 \frac{1}{\sqrt{9-4x^2}}\,dx$ (5 marks)

6. Find the equation of the tangent to the curve $2xy = 3$ at the point when $x = 3$. (5 marks)

7. Find the acute angle between the lines $3y - x - 6 = 0$ and $y - 2x + 4 = 0$. (5 marks)

8. Find the Cartesian equation of a curve whose polar equation is $r = 4\sin\theta$.

## SECTION B

9. (a) Find the distance of the point (2, 1, 2) from the line $x - 1 = y - 2 = Z - 3$. (7 marks)

(b) Find the position vector of the point of intersection of the planes $x - 2y - 2z = 0$, $2x + 3y + z = 1$ and $3x - y - 3z = 3$ (5 marks)

10. (a) Prove by induction:

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$ (6 marks)

(b) The fifth term of an AP is 25 and the fifteenth term is 75. Find the 10th term. (6 marks)

11. Express $\frac{3x^2+x+1}{(x-2)(x+1)^3}$ into partial fractions. Hence compute

$\int_3^4 \frac{3x^2+x+1}{(x-2)(x+1)^3}dx$ (12 marks)

12. (a) Solve for $\theta$ if $sin^2\frac{\theta}{2} = 2 + \cos\theta$ for $180^0 \leq \theta \leq 360^0$. (5 marks)

(b) Solve triangle ABC where C = 5.2 cm, a = 7.4 cm and angle B = $41^0$. (7 marks)

13. (a) Find the equation of the circle which passes through the points (1, 2), (2, 5) and (-3, 4). (5 marks)

(b) A and B are points (3, 0) and (-1, -3) respectively. P is a variable point such that angle APB is right angled. Find and sketch the locus of P. (7 marks)

14. Differentiate

(i) $cot^{-1}(Inx)$

(ii) $(\sin x)^x$ (12 marks)

15. Solve

(a) $x < \frac{2}{x-1}$ (5 marks)

(b) $\sqrt{3x+1} + \sqrt{4x+5} = \sqrt{16x+9}$ (7 marks)

16. (a) Find the general solution of

$$\frac{dy}{dx} = \frac{x^2 + y^2}{x(x+y)}$$

(b)The rate at which a radioactive material decays are proportional to the amount of such material present. Half of the original; mass M of the radioactive material undergoes disintegration in a period of 1500 years.

(i) What percentage of the original mass will remain after 3000 years?
(ii) In how many years will one tenth of the original mass remain?

**END.**

# PURE MATHEMATICS SET TWELVE (P425/1)

**INSTRUCTION TO CANDIDATES**

- Answer all the **eight** questions in section A and any **five** from section B.
- Any additional question(s) answered will not be marked
- All necessary working must be shown clearly
- Graph paper is provided
- Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

**SECTION A**

**Answer all questions in section A**

1. Use Demoivre's theorem to simplify the following (5 marks)

$$\frac{\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^5 \left(\cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi\right)^3}{\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^2}$$

2. Evaluate $\int_0^6 \frac{dx}{\sqrt{12x - 2x^2 - 9}}$ (5 marks)

3. Given that in the equation $ax^2 + bx + c = 0$ are roots of the equation 3 times the other show that $3b^2 = 16ac$. (5 marks)

4. Differentiate and simplify $f(x) = \sqrt{\frac{(1+x^2)^3}{2+x^2}}$ (5 marks)

5. Using series convert $0.6\bar{6}6$ into a fraction (5 marks)

6. Prove that sinA + sinB – sinC = $4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$ where A, B and C are angles of a triangle. (5 marks)

7. Show that the points A(1,2,3) B(-1,-2,-1) C(2,3,2) and D(4,7,6) are vertices of a parallelogram. (5 marks)

8. When the polynomial $3x^2 + ax^2 - bx + 1$ is divided by $(x-1)^2$ the remainder is $39x - 51$. Find the values of **a** and **b**. (5 marks)

## SECTION B

9. (a)In how many ways can letters of the word **PERMUTATION** be arranged? In how many of these are the T's together and how many of these are vowels together.

(b)A committee of six is to be selected from 9 women and 3 men

(i)In how many ways can a committee be formed to include at least ONE man on the committee?

(ii)In how many ways can a committee be selected to include at least 2 men and two women? (12 marks)

10. (a) Prove that acos2B + 2bcosAcosB = AcosB – bcosC

(b)Using Rsin (θ + B) form solve the equation 3sin$\theta$ - cosθ = 3 for values of θ from 0 to 2π (12 marks)

11. Given f(x) $= \frac{x^4 - 3x^3 + 2x^2 + x + 3}{(x^2+1)(x-2)(x-1)^2}$, partialise f(x) and hence intergrate $\int_0^3 f(x)dx$ (12 marks)

12. (a) Prove that $\frac{x-4}{1} + \frac{-y-3}{4} = \frac{1+z}{7}$ and $\frac{1-x}{-2} = \frac{1+y}{-3} = \frac{z+10}{8}$ intersect hence state the point of intersection.

(b) Show that the points with position vectors 4i – 8j – 13k, 5i – 2j – 3k and 5i + 4j + 10k are vertices of a triangle. (12 marks)

13. Find the equation of the normal to the parabola $y^2 = 4ax$ at the point $(ap^2, 2ap)$.

Show that the normal to the curve at L(a, 2a) passes through the point B(5a, - 2a) Prove that there are just one other point M on the curve at which the normal passes through B and determine the coordinates of M. (12 marks)

14. (a) Differentiate and simplify

(i) $y = \frac{e^{x^2}\sqrt{sinx}}{(2x+1)^3}$ (ii) $y = x^x (\cos x)^x$

(b) Use small change to find $\sqrt{98}$ correct your answer to one decimal place

(12 marks)

15.(a)Find the region where the curve $y = \frac{2x^2-ax+4}{x^2-2x+1}$ does not lie.

Hence determine the turning points and their nature.

(b)State the asymptotes and intercepts.

(c)Sketch the curve (12 marks)

16. (a)Expand $(1 - 3x)^{1/3}$ up to $x^3$ use your expression to find $\sqrt[3]{24}$ to 4 significant figures.

(b)A man pays a premium of 100 dollars at the beginning of every year to insurance company. After how many years will he accumulate more than 2270 dollars if the rate is 5% per year? (12 marks)

**END**

# APPLIED MATHEMATICS SET ONE (P425/2)



**INSTRUCTIONS TO CANDIDATES:**

- Answer **all** the **eight** questions in Section **A** and **five** questions from Section **B**.
- **All** working must be shown clearly
- Begin each answer on a fresh sheet of paper
- Graph paper is provided
- Silent, non-programmable calculators and mathematical tables with a list of formulae may be used
- In numerical work, take g to be $9.8ms^{-2}$

## SECTION A (40 MARKS)

1. Events **A** and **B** are such that $P(A \cup B) = \frac{7}{10}, P(A/B) = \frac{7}{12}$ and $4P(A) = 3P(B)$

Find the;

(i) $P(A \cap B)$ (03marks)

(ii) $P(A \cup \overline{B})$ (02marks)

2. Use the trapezium rule with 5 strips to estimate the value of $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin x}dx$.

Truncate your answer to 3 significant figures. (05marks)

3. A car of mass 1.05tonnes moves with a resultant force given by

$F = (2000 - \frac{x^2}{5})N$. Calculate the work done as the car travels a distance of 100m.

(05marks)

4. A random variable X is uniformly distributed over the interval a to 5. If X has a mean of 3, find :

(i) The value of **a** (02marks)

(ii) $P(2 < X < 3.5)$ (03marks)

5. A particle moving with an acceleration given by the expression

**a** = $3e^{-t}$**i** + 5cost**j** + 4sint**k** initially has the velocity **v** = 6**i** – 2**j** + 3**k**. Find the speed of the particle after 2 seconds. (05marks)

6. A random sample of 200 people were asked the length of time they spent in the shower, the last time that they took one. The results were as follows:
$\sum x = 909, \sum x^2 = 4555$.

(a) Calculate the unbraced population variance. (02marks)

(b) Determine the 97.5% confidence interval for the mean time spent in the shower. (03marks)

7. The table below shows the number x and its natural logarithm.

| x (X) | 1.23 | 1.79 | 2.04 | 2.62 |
|---|---|---|---|---|
| $\log_e x$ | 0.207 | 0.582 | 0.713 | 0.963 |

Use linear interpolation or extrapolation to estimate the value of

(i) x when $\log_e x = 0.811$ (03marks)

(ii) $\log_e x$ when $x = 1.14$ (02marks)

7. A block of mass 14kg is suspended from two strings. If the strings are inclined at $50^0$ and $60^0$ to the vertical, calculate the tensions in the strings.(05marks)

## SECTION B (60 MARKS)

8. (a) The life time of a transistor ratio battery is normally distributed with a mean of 160 hours and standard deviation of 30 hours, calculate the probability of batteries which have a life between 150 and 180 hour.(04marks)

(b) The chance that a cow is infected by a certain tick disease is $\frac{2}{5}$. If there are 150 cows in a certain form, find the 99.5% confidence limits for the mean number of cows that are infected. (08marks)

9. A transport plane A and a fighter plane B flying at the same altitude are initially 850.8 km apart with A on a bearing of N$36.22^0$W of B. If A is flying at 200kmh$^{-1}$ on a bearing of N$60^0$ E while B is flying at 450 kmh$^{-1}$ on a bearing of N$10^0$W.

(a) Show that if the planes maintain their velocities, then collision will occur. (05marks)

(b) Find the:

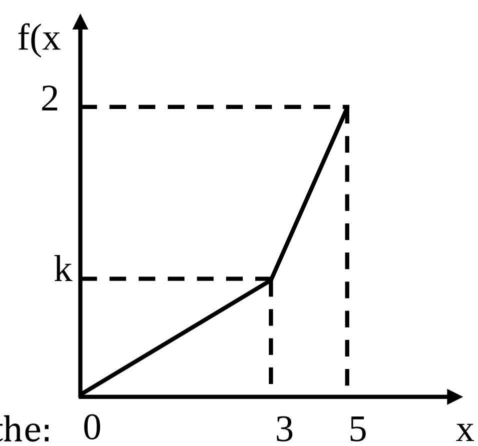
(i) time taken before collision occurs (04marks)

(ii) Distance that A flies before collision. (03marks)

10. (a) Using the same graph, show that the curves $y = 2e^x$ and $y = 4 - x^2$ have two real roots. (06marks)

(b) Using the Newton, rap son formulae twice, find the negative root of the equation $2e^x + x^2 = 4$, giving your answer correct to two decimal places. (06marks)

11. The probability density function of a continuous random variable X is represented graphically as shown below:




Find the:

(a) Value of **k** (03marks)

(b) Equations of $f(x)$ (03marks)

(c) $P(2 < x < 4)$ (03marks)

(d) Mean of X (03marks)

12. (a) A non-uniform rod AB of length 4m and mass 10kg is suspended horizontally from the ends of strings AC and BD. If AC and BD are inclined at $30^0$ and $50^0$ respectively with the vertical,

Calculate the:

*(i)* tension in the string BD (05marks)

*(ii)* Distance of centre of gravity of the rod from A. (02marks)

*(b)* A body of mass 8kg in contact with a plane inclined at $50^0$ is just prevented from sliding down the plane by a horizontal force **P**. If the angle of friction between the plane and the body is $25^0$. Calculate the magnitude of **P**. (05marks)

13. (a) Show that the simplest iterative formula based on Newton-Raphson Method for solving the equation $2\sin x - x = 0$ is given by

$\frac{2x_n - \tan x_n}{2 - \sec x_n}$, n = 0, 1, 2, … (03marks)

*(b)* Construct a flow chart that:

(i) reads the initial approximation (03 marks)

(ii) computes and prints the root of the equation correct to 2 decimal places after 3 iterations (05marks)

(iii) Taking the initial approximation as 1.8, perform a dry run of the flow chart. (04marks)

14. A particle A of weight 49N in contact with a horizontal table is connected by a light inelastic string passing over a smooth light pulley fixed at the edge of the table. The other end of the string carries another particle B of mass 2kg hanging freely. The system is released from rest and after 2 seconds, A collided and coalesced with a stationary particle of mass 0.1kg at rest on the ground. If the coefficient of friction between the table and the weight is 0.25,

a) Calculate the:

(i) acceleration of the system (04marks)

(ii) Tension in the string before collision. (02marks)

b) Find the change in kinetic energy of A immediately after collision. (06marks)

15. The age of people in a certain country in 2012 were as follows:

| Age (years) | 0-9 | 10-14 | 15-19 | 20-59 | 60-64 | 65-104 |
|---|---|---|---|---|---|---|
| Number of people in missions | 7.7 | 4.6 | 4.4 | 28.4 | 2.9 | 8.0 |

Calculate the:

(a) Mean, (03marks)

(b) Median, (03marks)

(c) Modal age (04marks)

(d) Number of people under 25 years. (02marks)

**END**

# APPLIED MATHEMATICS SET TWO (P425/2)

***INSTRUCTIONS TO CANDIDATES;***

- *Answer **ALL** the eight questions in Section **A** and any **FIVE** from Section **B**.*
- *Any additional question will not be marked.*
- *All necessary working must be shown clearly.*
- *Begin each answer on a fresh sheet of paper.*
- *Graph paper is provided.*
- *Silent non programmable scientific calculators and Mathematical tables with a list of formulae may be used.*
- *In numerical work, take acceleration due to gravity g to be 9.8ms-*

## SECTION A

1. The displacement of a body after time t is given by $r(t) = 2\sqrt{3}\sin t\,i + 8\cos t\,j$. Find the speed of the body when $t = \frac{\pi}{6}s$. *(5 marks)*

2. Use the trapezium rule with 6 ordinates to find an appropriate value for $\int_0^{\frac{\pi}{2}} x\sin x dx$ correct to 3 decimal places.

3. Two events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{8}$ and $P(A/B) = \frac{7}{8}$. Find

   (i) $P(A \cap B)$

   (ii) $P(B/\overline{A})$

4. Given the following values for the time (t) taken to cover a distance ($x$) by a Gaaga bus as

| | | | | | |
|---|---|---|---|---|---|
| Distance (x/m) | 0 | 5 | 10 | 15 | 20 |
| Time (t/m) | 0 | 12 | 25 | 39 | 54 |

Use linear interpolation to estimate

(i) $t$ when x = 13m

(ii) $x$ when t = 42 minutes

5. A random variable X *takes* on values $x$ and is such that $P(X=0)=0.1$, $P(X=1)=0.4$ $P(X=2)=0.2$ and $P(X=3)=0.3$

Find (i) $E(X)$ (ii) Var (X)

6. A box of mass 2kg lies on a rough horizontal floor, coefficient of friction 0.5. A light string is attached to the box inclined at 30° above the horizontal in order to pull the box across the floor. Calculate the tensional value that must be exceeded for motion to occur. *(5 marks)*

7. Past data suggest that for every 100 students applying to university, only 20 students are admitted. Find the probability that from the next 200 applicants received, the university will admit between 44 students and 56 students.

8. Forces $F_1$, $F_2$ and $F_3$ are given as $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$N, $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$N and $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$N and act at points with position vectors. (1, 2)m, (-2,3)m and (3, 2)m respectively. Show that the forces reduce to a couple. *(5 marks)*

## SECTION B

9. A random variable X has its pdf given by

$$f(x) = \begin{cases} \lambda x^2 & 0 \le x \le 2 \\ \lambda(8-2x) & 2 \le x \le 4 \\ 0 & otherwise \end{cases}$$

(i) Sketch *f(x)*

(ii) Find the value of $\lambda$

(iii) Determine $E(3x - 2)$

(iv) Find $P(X > 3)$

10. The table below shows marks obtained by S.6 students from NSSN.

| Marks | No. of students |
|---|---|
| $< 30$ | 3 |
| $< 35$ | 8 |
| $< 45$ | 12 |
| $< 50$ | 18 |
| $< 65$ | 11 |
| $< 75$ | 5 |
| $< 80$ | 2 |
| $< 90$ | 1 |

(a) Determine the mean and standard deviation of the marks

(b) Draw an ogive for the data and use it to estimate the minimum mark for a distinction if the 10% of the students obtained distinction. *(12 marks)*

11. At $t = 0$, the position vectors $\boldsymbol{r}$ and velocity vectors $\mathbf{V}$ of particles A and B are given as follows.

$r_A = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$m $\qquad V_A = (-6\boldsymbol{i} + \boldsymbol{k})\ ms^{-1}$

$r_B = (\mathbf{4i} - 14\mathbf{j} + \mathbf{k})$ m $\qquad V_B = (-\mathbf{5}\boldsymbol{i} + \boldsymbol{j} + 7\boldsymbol{k})ms^{-1}$

Find

(i) The position vector of B relative A at a time t seconds.

(ii) The value of t when A and B are closest together.

(iii) The least distance between A and B.

(iv) The position of A from the origin at the time when A and B are closest together. *(12 marks)*

12 (a) The sample below was taken from a normal distribution. 101, 107, 102, 104, 106,100, 108 and 104. Find the 90% confidence interval for the population mean. *(7 marks)*

(b) The mean weight of tilapia in fish farm is 980g and the standard deviation 100g. What is the probability that a catch of 10 tilapia will have a mean weight per fish between 910g and 1050g? *(5 marks)*

13 (a) Show that the root of the function $f(x) = \sin x + x - 1 = 0$ lies between 0 and 1. (3 *marks)*

(b) Find the Newton Raphson formula for solving the equation in (a) above and hence draw a flow chart that.

(i) reads initial approximation $x_0$

(ii) Computes and prints the root, correct to 3 decimal places.

(c) Perform a dry run for the flow chart. *(9 marks)*

14. A uniform ladder AB of weight 50N and length 8m is freely hinged at A to a vertical wall and carries another weight of 30N at end B. The ladder is held horizontally in equilibrium by a string which has one end attached to B and the other end attached to a point C on the wall 4m above A.

Determine;

(i) The tension in the string

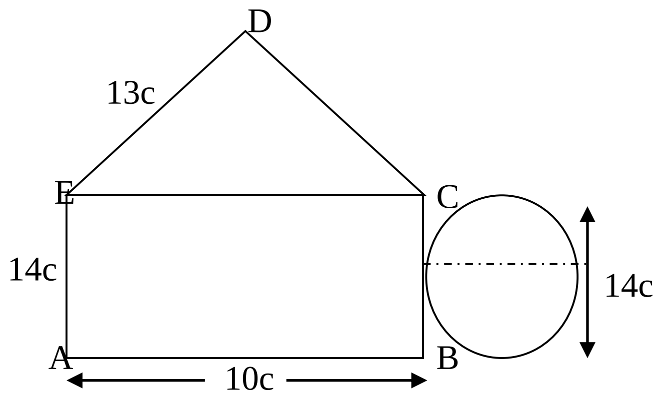
(ii) The magnitude of the reaction at the hinge and its direction. *(12 marks)*

15 (a) The numbers A and B are approximately by a and b with respective errors $\Delta a$ and $\Delta b$. Derive an expression in obtaining the relative error in AB showing all the precautions taken. *(4 marks)*

(b) If $P = \frac{15.36 + 27.1 - 1.672}{5.62 - 2.4}$. Determine the range with which P lies and hence find the absolute error in P.

16 (a) Masses of 5kg, 2kg, 1kg and 3kg are situated at (3, 1)m, (4, 3)m, (5,2)m and (-3,1)m respectively. Determine the coordinates of the centres of gravity of the masses. *(5 marks)*

(b) The figure below shows a composite lamina consisting of a rectangle, triangle and a circle.




Determine the coordinates of the centre of gravity of the composite lamina.

**END**

# APPLIED MATHEMATICS SET THREE (P425/2)

**INSTRUCTIONS TO CANDIDATES:**

- *Attempt **all** the questions in section A and any **five** questions from section B.*
- *All necessary calculations must be done on the **same page** as the rest of the answers. Therefore there is no paper for rough work.*
- *Mathematical tables with a list of formulae and squared papers are provided.*
- *Silent, non-programmable scientific calculators may be used.*
- *In numerical work, take **g** to be 9.8 $ms^{-2}$.*

**SECTION A: (40 MARKS)**

*Answer **all** questions in this section*

**(c)** Particles of masses 4kg, 3kg, 2kg, 9kg and 2kg are placed at points with position vectors: $2\mathbf{i} + 4\mathbf{j}$, $-3\mathbf{i} - 2\mathbf{j}$, $\mathbf{i} - 4\mathbf{j}$, $-3\mathbf{i} + 5\mathbf{j}$ and $4\mathbf{i} + 3\mathbf{j}$ respectively. Find the position vector of centre of gravity of the system of particles. ***(5 marks)***

2. Box A contains 4 yellow and 3 green oranges while box B contains 5 yellow and 7 green oranges. An orange is randomly selected from A and placed in B and then an orange is randomly selected from B and put back in A. If X is the number of yellow oranges remaining in box A after the operation,

(i) Draw a probability distribution table for X and show that X is a random variable,

(ii) Find the mean of X. ***(5 marks)***

3. A car travelling along a straight road covers consecutive distances 1km and 2km in equal time interval of 10 minutes. Find the;

(i) Acceleration of the car,

(ii) Initial velocity. ***(5 marks)***

4. By drawing a suitable graph, determine the positive root of the equation $2x^2 - x - 5 = 0$. ***(5 marks)***

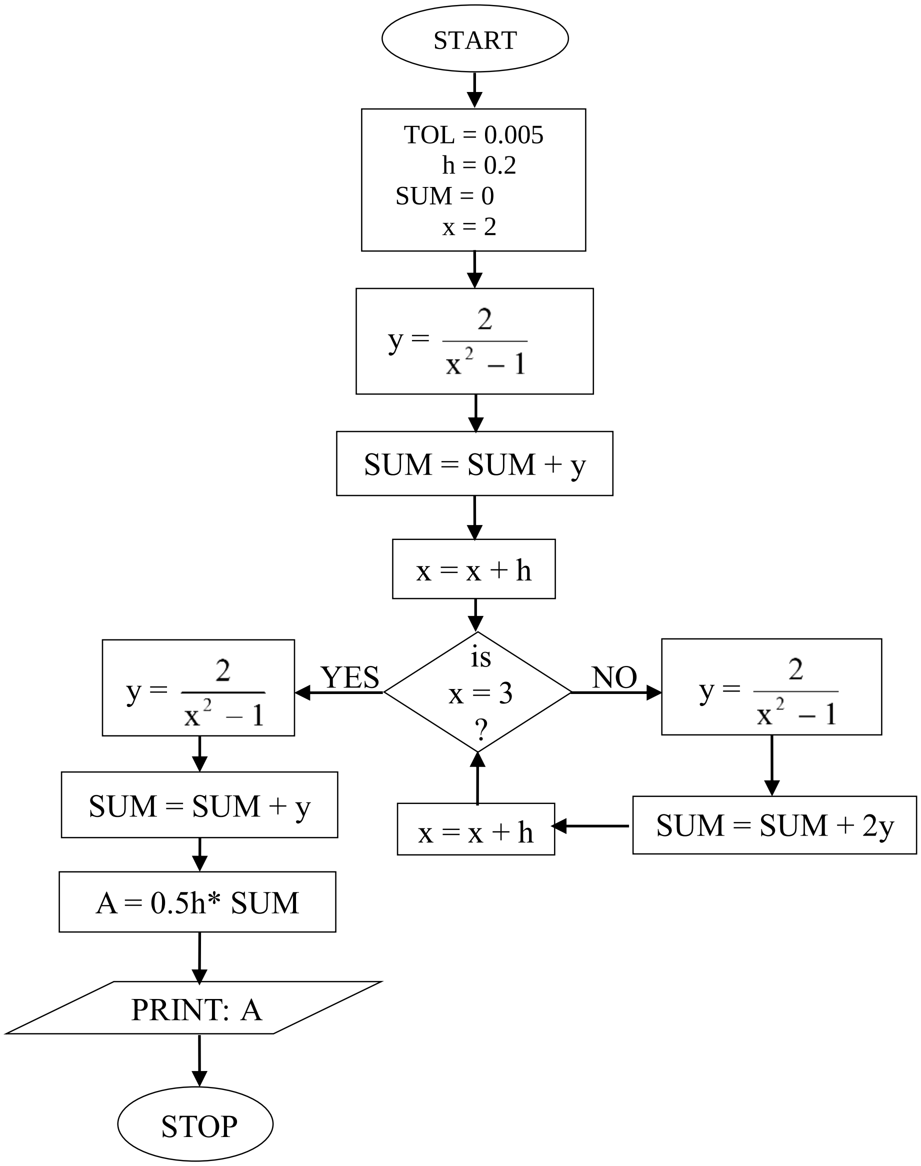
5. Marks obtained by students in a certain test are uniformly distributed with mean 50 marks. If the least mark is 40, calculate the probability that a student chosen at random has a mark between 47 and 53. ***(5 marks)***

6. A bullet of mass 100 grams travelling at $400ms^{-1}$ horizontally is fired into a block of mass 20kg at rest on a horizontal table. If the coefficient of friction between the table and the block is 0.2, calculate the

(i) Acceleration of the block if the bullet is embedded in the wood,

(ii) Distance moved by the block. ***(5 marks)***

7. Study the flow diagram below.




Perform the dry run of the flow chart and state its purpose. ***(5 marks)***

8. The table below shows the height (cm) and the age (years) of a group of students.

| | | | | | | |
|---|---|---|---|---|---|---|
| Height (cm) | 120 | 152 | 90 | 165 | 152 | 144 |
| Age (years) | 16 | 17 | 10 | 17 | 14 | 15 |

Calculate the rank correlation coefficient between height and age and comment on the relationship. ***(5 marks)***

## SECTION B (60 MARKS)



***Answer five questions in this section. All questions in this section carry equal marks.***

9. (a) A particle performs simple harmonic motion between two points A and B which are 4m apart with point O as center. When the particle is at point 1 m from O, its speed is $\frac{1}{\sqrt{3}}$ $ms^{-1}$. Calculate the

(i) Acceleration when the particle is 0.4m from A,

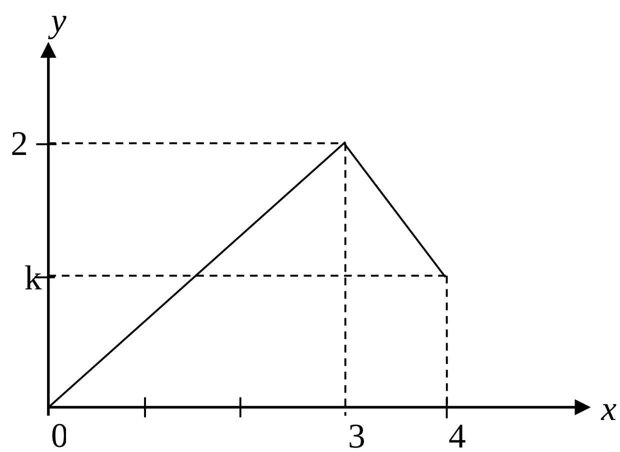
(ii) Displacement of the particle after 2seconds. ***(12 marks)***

(b) A particle of mass 2kg is freely suspended from the end B of a light elastic string of natural length 1m and the other end of the strong is fixed to a point A. When in equilibrium at a point E, AE = 1.5m. The particle is then pulled vertically downwards to a point C, 0.2m from E and released.

(i) Show that the subsequent motion is simple harmonic.

(ii) Find the time taken by the particle to reach a point 0.15m from C for the first time. ***(12 marks)***

10. The probability density function of a continuous random variable X is represented graphically as shown.




Find the

(i) Value of k,

(ii) Equations of the p.d.f. f(x),

(iii) P(X>2/X<3.5). ***(12 marks)***

11. (a) Given that the exact quantities A and B are approximated using a and b respectively with corresponding errors $\Delta A$ and $\Delta B$, show that the maximum possible error in $\frac{a}{b}$ is $\left|\frac{a}{b}\right|\left(\left|\frac{\Delta A}{a}\right|+\left|\frac{\Delta B}{b}\right|\right)$.

(b) The quantities x = 14.57, y = 17.9112 and z = 4.3 are each rounded off to the given number of decimal places. If $d = \frac{x - y}{y - z}$, find

(i) Range of values within which d lies correct to four decimal places,

(ii) Percentage error in d correct to two significant figures. ***(12 marks)***

12. The ages of students attending a certain function are normally distributed with mean 18 years. If 85.1% of the students have their ages greater than 17 years, find the probability that

(a) A student selected at random has an age between 16 and 19 years.

(b) A random sample of 16 students have their mean ages between 17.5 and 18.7 years. ***(12 marks)***

13. The points A, B, C and D are the vertices of a square of side 4m. Forces of magnitudes 8N, 3N, $4\sqrt{2}$ N, 6N and 3N act along the sides BA, CB, AC, DA and DC respectively, the direction of the forces being indicated by the order of the letters. Find the;

(a) Magnitude of the resultant force.

(b) Equation of line of action of the resultant force taking AB and AD as the horizontal and vertical axes respectively.

(c) Distance from A where the line of action of the resultant crosses AB.

***(12 marks)***

14. Derive the simplest iterative formula based on Newton Raphson method that can be used to find the cube root of a number N.

(a) By drawing a suitable graph, find the value of $\sqrt[3]{12}$ .

(b) Using the value from the graph as an initial approximation, find $\sqrt[3]{12}$ correct to three decimal places. *(12 marks)*

15. The table below shows the weight of seeds of a certain type of plant.

| W (grams) | $< 0.10$ | $< 0.25$ | $< 0.35$ | $< 0.50$ | $< 0.60$ | $<0.65$ | $< 0.80$ |
|---|---|---|---|---|---|---|---|
| Frequency | 2 | 3 | 5 | 9 | 3 | 2 | 3 |

Calculate the

(i) Mean weight,

(ii) Standard deviation weight.

(iii) Draw a histogram and use it to estimate the modal weight. *(12 marks)*

16. (a) A car of mass 1500kg travelling along a horizontal road at a maximum power has a maximum speed of $150kmh^{-1}$. If there is a resistance of 60N, find the

(i) Maximum power,

(ii) Acceleration when the car is travelling at $72kmh^{-1}$ if power remains constant.

(b) A hammer of mass 4.5kg falls through a vertical height of 1m and hits a nail of mass 50 grams directly without rebounding. If the nail is driven into a piece of wood a depth of 2cm, find average resistance to penetration assuming that it is constant. *(12 marks)*

**END.**

# APPLIED MATHEMATICS SET FOUR (P425/2)

**INSTRUCTIONS TO CANDIDATES:**

- *Attempt **all** the **eight** questions in section A and any **five** questions from section B.*
- *All necessary calculations must be done on the **same page** as the rest of the answers. Therefore there is no paper for rough work.*
- *Mathematical tables with a list of formulae and squared papers are provided.*
- *Silent, non-programmable scientific calculators may be used.*
- *In numerical work, take g to be 9.8 $ms^{-2}$.*
- *States the degree of accuracy at the end of the answer to each question attempted using a calculator or table and indicate **Cal** for calculator, or **Tab** for mathematical tables.*

## SECTION A

1. A group of students participated in a competition and two judges awarded them marks as follows.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Judge I | 40 | 59 | 84 | 30 | 66 | 72 | 91 |
| Judge II | 40 | 60 | 89 | 44 | 85 | 77 | 85 |

Calculate the rank correlation coefficient between the judges and comment on the relationship. (5mrks)

2. A bag of weight 40N placed on a plane inclined at $30^o$ above the horizontal is just about to move up the plane when a force F parallel to the plane is applied on it. If the coefficient of friction between the bag and the plane is 0.125, calculate the magnitude of F. (5 marks)

3. The table below shows x and the function f(x)

| | | | | |
|---|---|---|---|---|
| x | 50.24 | 48.11 | 46.93 | 44.06 |
| f(x) | 4.116 | 7.621 | 9.043 | 11.163 |

Use linear interpolation or linear extrapolation to find the value of

(i) x when $f(x) = 8.614$, (3 marks)

(ii) $f(51.07)$. (2 marks)

4. Events A and B are such that $P(B) = \frac{7}{20}$ and $P(\overline{A}/B) = \frac{3}{7}$. Find

(i) $P(A \cap B)$, (3 marks)

(ii) $P(A/B)$. (2 marks)

5. A game is played by tossing three fair tetrahedral dice at once. If all the bottom faces show the figure 4, the player wins 10,000/=; If two faces show the figure 4, the player wins 5,000/= and if a face shows a 4, the player wins 2,000/= otherwise the player gets nothing. Find the

(i) Probability that a player wins at least 5,000/= after playing the game once.

(3 marks)

(ii) Expected amount of money that a player can win. (2 marks)

6. Two airports A and B are 300 km apart with B on a bearing of $150^0$ from A. A wind of 36 $kmh^{-1}$ blows from $030^0$. If the velocity of the wind remains constant and an aircraft whose speed in still air is 250 $kmh^{-1}$ is to be flown from A to B, calculate thetime of flight. (5 marks)

7. The sides of a cuboid $x = 12.55$cm, $y = 4.25$cm and $z = 8.2$cm are each measured correct to the given number of decimal places. Calculate the limits within which the volume of the cuboid is expected to lie, correct to two decimal places. (5 marks)

8. A body of mass 0.5kg suspended from a fixed point A by a light elastic string of natural length 4cm and modulus of elasticity 19.6N hangs vertically. If the body is pulled vertically downwards to a below equilibrium position and then released, it just reaches the level of A, calculate the extension in the string before the body is released. (5 marks)

## SECTION B

9. The probability density function of a continuous random variable X is represented by the equations below.

$$f(x) = \begin{cases} \frac{2}{13}(x+1) \; ; & 0 \le x \le a, \\ \frac{2}{13}(5-x) \; ; & a \le x \le b, \quad 0 \; ; \quad \text{Otherwise.} \end{cases}$$

Calculate

(a) The values of a and b. (9 marks)

(b) $P(X < 2.5)$, (3 marks)

10.(a) A particle of mass 40 grams performs simple harmonic motion about point O between two points A and B on a horizontal plane. If $AB = 10$cm and the period of motion is $\frac{1}{10}\pi$ seconds, find the work done by the particle in moving from O to A. (4 marks)

(b) A block of mass 2.5 kg is suspended from the end O of a light inelastic spring of natural length 0.5m. The block rests in equilibrium at a point B vertically below O such that $OB = 0.75$m. If the block is pulled a further distance of 0.25m below B and released.

(i) Show that the motion is simple harmonic, (4 marks)

(ii) Find the time taken for the particle to move directly from C to B where $BC = 0.125$m. (4 marks)

11. The able below shows the distribution of marks of students in a test.

| Score | Frequency |
|---|---|
| $20 \le x < 30$ | 4 |
| $x < 45$ | 3 |
| $x < 50$ | 9 |
| $x < 65$ | 21 |
| $x < 75$ | 3 |
| $x < 80$ | 5 |
| $x < 100$ | 14 |



(a) Draw a histogram and use it to estimate the modal mark.(5 marks)

(b) Calculate the

(i) mean score, (3 marks)

(ii) Standard deviation score, (4 marks)

12. A square ABCD of side 4m has forces of magnitude 8N, 3N, 3N, 4N and $2\sqrt{2}$ N acting along AB, CB, DA, CD and BD respectively. Taking AB and AD as x and y axes respectively,

(a) Find the distance from A where the line of action of the resultant crosses AB.

(9 marks)

(b) When a couple of magnitude M is introduced, the force system is reduced to a single force passing through B. Find M and its direction. (3 marks)

13. The exact numbers A and B have been estimated using a and b respectively. Given that $\Delta A$ and $\Delta B$ are the corresponding errors,

(a) Show that the absolute error in $\frac{A}{B}$ is given by $\left|\frac{a}{b}\right|\left(\left|\frac{\Delta A}{a}\right| + \left|\frac{\Delta B}{b}\right|\right)$ (5 marks)

Hence deduce the expression for error in $\frac{A-B}{C}$. (3 marks)

(b) The numbers a = 4.314, b =18.92 and c = 15.0214 are each rounded off to the given number of decimal places. Use the expression in (a) to calculate the percentage error in $\frac{a - b}{c}$ correct to two significant figures. (4 marks)

14. (a) A block of mass 10 kg suspended from the end A of a light inelastic string of length 50cm is kept in equilibrium when a force F inclined at $60^0$ to the downward vertical acts on it. If the string is displaced through 30cm horizontally from the vertical through A, calculate the magnitude of F. (5 marks)

(b)A rod AB of mass 5 kg and length 6m rests with the end A on a rough horizontal ground and B against a rough vertical wall. If coefficients of frictions at A and B are 0.25 and 0.2 respectively and the rod is inclined at $30^0$ to the horizontal, find the distance from A of a point on the rod where the weight acts. (7 marks)

15. The probability that a marksman aims and hits a target with a single shot is 0.4. If the marksman is given 50 bullets, find the probability that he hits the target:

(i) Exactly 24 times, (7 marks)

(ii) Between 18 and 27 times inclusive. (5 marks)

16.(a) Use trapezium rule with 6 sub intervals to estimate the value of $\int_{0.5}^{1.0} \frac{x^2}{1+x^2}\,dx$ correct to **four** decimal places. (6 marks)

(b) Calculate the percentage error in using the trapezium rule to estimate the value of the integral in (a) above correct to **two** significant figures. (6 marks)

# APPLIED MATHEMATICS SET FIVE (P425/2)



**INSTRUCTIONS TO CANDIDATES:**

- Answer **all** the **eight** questions in section **A** and any **five** from section **B.**
- Any additional question (s) answered will **not** be marked
- All necessary working **must** be shown clearly
- Begin each answer on a fresh sheet of paper
- Graph paper is provided
- Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.
- In numerical work, take **g to be 9.8 ms$^{-2}$.**

**SECTION A (40 MARKS)**

1. Events A and B are such that
P(A) = 0.8, P (A / B) = 0.8 and P(A∩B) = 0.5.
Find;
(i) P(B) (02 marks)
(ii) P(A ∩ B / A ∪B ) (03 marks)

2. A particle of mass **2kg** moves with velocity $\boldsymbol{e^t i + 2e^{-2t} j - \text{sin}t\, k}$. Find the power developed after **4** seconds. (05 marks)

3. If **p = 4.7, q = 80.00** and **r = 15.900** are rounded off with corresponding percentage errors of 0.5, 0.5 and 0.05, calculate the relative error in the expression $\frac{pq}{r}$ correct to 2 significant figures. (05 marks)

**4.** In a survey, 200 people were asked the length of time that they spent in the shower, the last time that they took one. The results were as follows; $\boldsymbol{\Sigma x = 909}$, $\boldsymbol{\Sigma x^2 = 4555}$.
(a) Find an unbiased estimate of the population variance. (02 marks)
(b) Determine the **97.5%** confidence interval for the mean time spent in the shower. (03 marks)

5. A particle of weight **78.4N** is released from rest at the top of a plane inclined at **30$^0$** to the horizontal. If the coefficient of friction between the particle and the plane is **0.2**, find the
(i) acceleration (03 marks)
(ii) Velocity after covering 10m. (02 marks)

6. The following scores were obtained during the sports day competition by different students' houses in a certain school.;

**42.2, 44.6, 47.5, 42.6, 51.4, 53.7, 56.8, 42.2, 59.2** and **61.7.**

Find the;

(i) mean score (02 marks)

(ii) variance (03 marks)

7. By using the Newton Raphson formula and $x_0 = \pi/2$ as the initial approximation to the root of the equation $3\sin x - 2x = 0$, show that the second approximation to the root is **1.5.** (04 marks)

8. A non-uniform rod **AB** of mass **20kg** and length **4m** is suspended horizontally from the ends of the strings **AC** and **BD** of $60^0$ and $45^0$ respectively with the vertical. If the tension in **AC** is **60N,** calculate the;

(i) tension in the string **BD** (03 marks)

(ii) Distance from **A** where the weight of the rod acts. (03 marks)

## SECTION B (60 MARKS)

**9.** The germination time of a certain species of beans is known to be normally distributed. In a given bath of these beans, **20%** take more than **6 days** to germinate and **10%** take less than **4 days.**

(i) Determine the mean and standard deviation of the germination time. (08 marks)

(ii) Find the **99.5%** confidence limits of the germination time. (04 marks)

10. (a) To a pilot of a plane flying at $180kmh^{-1}$ on a bearing of $S\ 30^0W$, the wind appears to blow from $S40^0W$ at $190kmh^{-1}$. Find the true speed of the wind. (04 marks)

(b) Two birds, A and B are initially at points with position vectors $(5\boldsymbol{i} + 8\boldsymbol{j} + 12\boldsymbol{k})\boldsymbol{m}$ and $(2\boldsymbol{i} - 4\boldsymbol{j} + 15\boldsymbol{k})\boldsymbol{m}$ respectively. if they are respectively flying with constant velocities of $(2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})\ ms^{-1}$ and $(\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k})ms^{-1}$, find the;

(i) time at which they are closest together (06 marks)

(ii) Distance that then separates them. (02 marks)

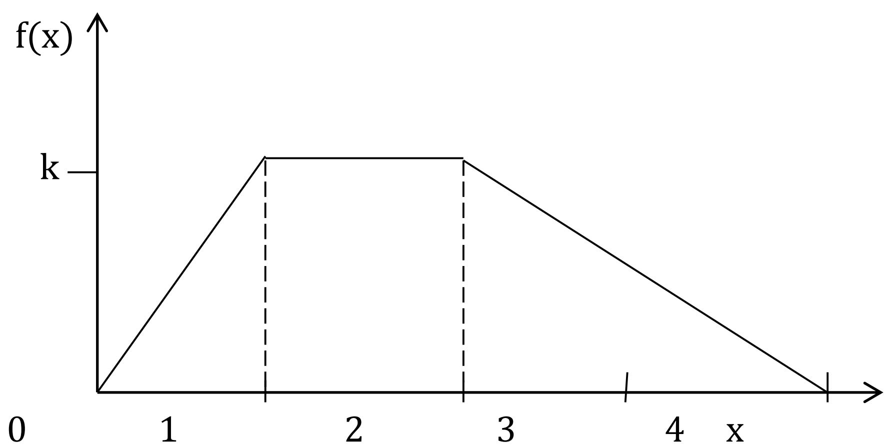
11. Show that the simplest iterative formula based on Newton Raphson method for solving the equation $\mathbf{2x^3 + 5x = 8}$ is

$$x_n + 1 = \frac{4\,x_n^3 + 8}{6\,x_n^2 + 5}, \text{ n} = 0, 1, 2, \ldots.. \quad \text{(03 marks)}$$

(a) Draw a flow chart that:
(i) Reads the initial approximation.
(ii) Computes and correct to three decimal places together with the number of iterations. (05 marks)

(b) Taking **1.5** as the initial approximation perform a dry of the flow chart. (04 marks)

12. The probability distribution function of a continuous random variable **X** is represented as shown.




Find the;
(i) Value of **k** (03 marks)
(ii) Expression for the distribution (03 marks)
(iii) $\mathbf{P(2.5 < x < 3)}$ (03 marks)
(iv) Mean of **X.** (03 marks)

13. Forces of magnitudes **7N, 6N, 10N, 13N and 15N** act along the lines **BA, BC, DC, DA and AC** respectively of the rectangle **ABCD** whose sides **AB =3*a*** and **BC = 4*a*** units. Find the;
(a) The magnitude and direction of the resultant force. (09 marks)
(b) Distance from **A** at which its line of action cuts **AD.** (03 marks)

14. (a) Locate each of the two roots of the equation $e^x - 4\sin x = 0$ in the interval **x= 0** and **x = 1.5** (04 marks)

(b) A motorist rides from Mbarara to Kyazanga, a distance of **80km.** If he leaves Mbarara at 8:00am and reaches distances **20km, 50km, 70km** at **8:30am, 9:00am, 9:40am** respectively.

(i) Find the approximate time he arrives at Kyazanga. (03 marks)

(ii) One day, at **9:35am** his car tyres burst and had to hire a lorry to carry the car to Kyazanga and was charged shs. **1000/=** per km. Find how much he paid for the hire. (05 marks)

15. The table below shows the ages of people who attended a certain function.

| Age (years) | frequency |
|---|---|
| 10 – 19 | 6 |
| 20 – 34 | 16 |
| 35 – 44 | 27 |
| 45 – 64 | 39 |
| 65 – 79 | 18 |
| 80 – 89 | 8 |

(a) Draw a cumulative frequency curve and use it to estimate the semi – interquatile range. (06 marks)

(b) Calculate the;

(i) Mean (03 marks)

(ii) standard deviation (03 marks)

16. A car of mass **1200kg** pulls a trailer of mass **300kg** up a slope of **1 in 100** against a constant resistance of **0.2N** per kg. Given that the car moved at a constant speed of **$1.5ms^{-1}$ for 5 minutes**, calculate the;

(i) Tension in the tow bar. (05 marks)

(ii) Work done by the car engine during this time. (03 marks)

(iii) A car has an engine that can develop **15kw**. If the maximum speed of the car on a level road is **120kmh$^{-1}$,** calculate the total resistance at this speed. (04 marks)

**END**

# APPLIED MATHEMATICS SET SIX (P425/2)

**INSTRUCTIONS TO CANDIDATES**

- Answer all the **EIGHT** questions in section **A** and any **FIVE** from section **B**
- Any **additional question(s)** answered will not be marked
- All necessary working **must** be shown **clearly**.
- Begin each answer on **a fresh** sheet of paper
- Squared paper is provided
- Silent , non –programmable scientific calculators and mathematical tables with a **list of formulae** may be used
- In numerical work; take acceleration to gravity **'g'** to be **9.8ms$^{-2}$**

**SECTION A (40MARKS)**

Attempt **ALL** questions from this section

1. A sample of **n** members of a given rotary club was asked how many crates of beer they took in a given month. The results were as follows

$\sum x = 225, \sum x^2 = 1755$. Find the possible values of **n** if the standard deviation is **1.5.** **(05 marks)**

2. Given that $X \sim Bin(4, P)$ and $P(x = 4) = 0.0256$. Find $P(x = 2)$. **(05 marks)**

3. A box of mass **5kg** is at rest on a plane inclined at **30$^0$** to the horizontal. The coefficient of friction between the box and the plane is $\frac{2}{5}$. What minimum force parallel to the plane would move the box up the plane. **(05marks)**

4. Use the trapezium rule with **five strips** to evaluate the integral:

$\int_0^1 2^{\sqrt{x}} dx$ to **two** decimal places. **(05marks)**

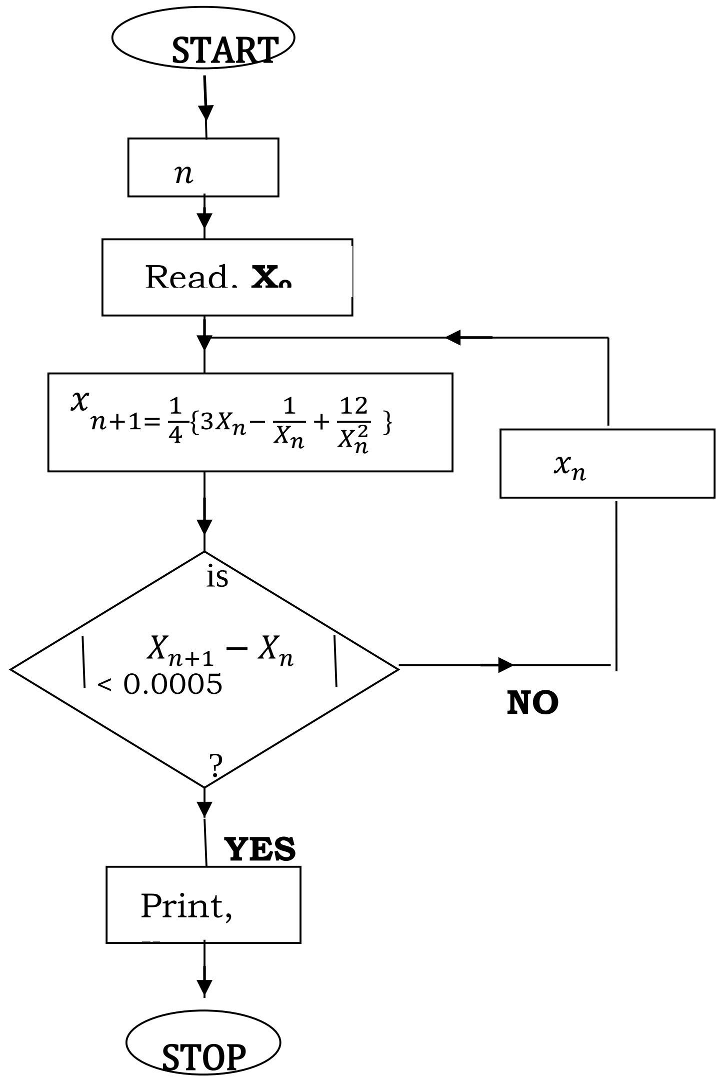
5. Two events **A** and **B** are such that $\boldsymbol{P(A/B) = \frac{2}{5}}$, $\boldsymbol{P(B) = \frac{1}{4}}$ and $\boldsymbol{P(A) = \frac{1}{5}}$. Find

(a) $P(A \cap B)$

(b) $P(A \cup B)$ **(05 marks)**

6. Three forces $\boldsymbol{i} + 5\boldsymbol{j} - 4\boldsymbol{k}, -3\boldsymbol{j} + 7\boldsymbol{k}$ *and* $3\boldsymbol{i} + 2\boldsymbol{k}$, act on a body from point $\boldsymbol{A(4, -2, 3)}$ to point $\boldsymbol{B(8, 4, 5)}$. Find the work done on the body. **(05 marks)**

7. The flow chart below was designed to solve the equation $\boldsymbol{f(x) = 0}$ for a given function $\boldsymbol{f(x)}$




Perform a dry run for $\boldsymbol{X_0 = 2.0}$ and state the function **f(x)** for which the chart was designed to be solved. **(05 marks)**

8. A particle of mass **4kg** is placed on a smooth plane whose height is **4m** and length **20m**. The particle is connected by a light string passing over a smooth pulley at the top of the plane to a mass of **3kg** hanging freely. Find the common acceleration and the tension in the string. **(05 marks)**

**SECTION B (60 MARKS)**

Answer only **five** questions from this section.

9. The age distribution of the applicants for a job is recorded in the table below.

| Age (Years) | 20- | 35- | 40- | 50- | 55- | 60- | 70- |
|---|---|---|---|---|---|---|---|
| No. of applicants | 12 | 10 | 7 | 8 | 9 | 4 | 0 |

(a) Calculate ;

(i) the mean **(04 marks)**

(ii) the upper quartile **(02 marks)**

(b) Draw a histogram for the data and use it to estimate the modal age. **(06 marks)**

10. (a) A uniform beam **AB** of weight **30N** is suspended by two strings at **A** and **B**. The beam is in equilibrium when the strings at **A** and **B** make angles $30^0$ and $60^0$ with the beam respectively. Find the tensions in the strings. **(05 marks)**

(b) A rectangular uniform lamina **ABCD** has sides $AB = 4a$ and $AD = 3a$. The corner at $D$ is folded so that $AD$ is along the side $AB$. A square of side **a** is removed from the corner $B$. Find the distance of the center of gravity of the resulting body from $AB$ and $BC$. **(07 marks)**

11. (a) By plotting graphs of $y = 1 - e^x$ and $y = 2^x$, show that the equation

$1 - e^x = 2$ has a root between $-1.5$ and $0$. Give the root to **two** decimal points. **(08 marks)**

(b) Use linear interpolation to approximate the root in (a). Correct your answer to **three** decimals places. **(04 marks)**

12. **OABC** is a square of side **OA= 2cm**. Forces of magnitude **2N, 8N, 4N, 4N** and **5√2N** act along $\overrightarrow{AO}$ , $\overrightarrow{AB}$ , $\overrightarrow{BC}$ , $\overrightarrow{CO}$ *and* $\overrightarrow{OB}$ respectively.

Find: (a) the magnitude and direction of the resultant force.

(b) The equation of the line of action of the resultant and hence the distance from **C** where the resultant crosses **CB**

13. Batteries for a transistor radio have a mean life under normal usage of, **160hours**, with a standard deviation of **30hours**. Assuming the battery life follows a normal distribution.

(a) Calculate the percentage of batteries which have life between **150hours** and **180 hours.** **(04marks)**

(b) Calculate the range, symmetrical about the mean, within which **75%** of the battery lives lie. **(03 marks)**

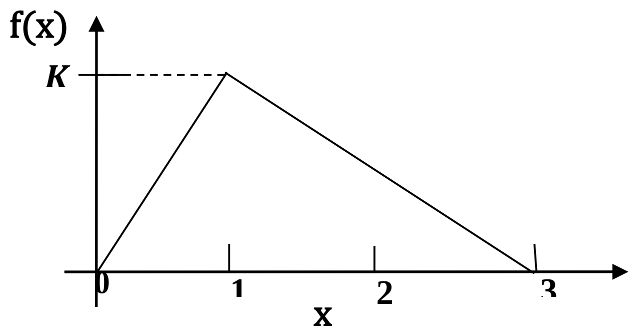
(c) If a radio takes four of these batteries and required all of them to be working, calculate the probability that the radio will run for at least **135 hours**. **(05 marks)**

14. (a)Two numbers **A** and **B** have maximum possible errors $\boldsymbol{e_a}$ *and* $\boldsymbol{e_b}$ respectively.

(i) Derive an expression for the maximum relative error in their product hence deduce the maximum possible error if **A= 2.03** and **B=1.547.** **(08 marks)**

(b) Find the limits within which the exact value of $\frac{\mathbf{7.684+0.31}}{\mathbf{7.684\ x\ 0.31}}$ lies. **(04marks)**

15. A continuous random variable $\boldsymbol{X}$ has a probability density function $\boldsymbol{f}(\boldsymbol{x})$ given by $\boldsymbol{f}(\boldsymbol{x}) = \mathbf{0}$ for $\boldsymbol{x} < 0$ and $\boldsymbol{x} > 3$ and between **0** and **3** its form is as shown below.




(a) Find the value of **k**

(b) Obtain $\boldsymbol{f}(\boldsymbol{x})$ and hence $\boldsymbol{E}(\boldsymbol{x})$

(c) Find the **cumulative** distribution function $\boldsymbol{F}(\boldsymbol{x})$

16. At **10.00am**, ship **A** moving at **20km/hr** due east is **10km** South East of another ship***B***. If ***B*** is moving at **14km/hr** in direction **S30⁰W** and the ships maintain their velocities, find the;

(a) Time when the ships are **closest** together and the **shortest distance** between the ships.

(b) Bearing of **A** from ***B*** at that time. **(12 marks)**

# APPLIED MATHEMATICS SET SEVEN (P425/2)



**INSTRUCTIONS TO CANDIDATES:**

- Answer **all** the **eight** questions in section **A** and any **five** questions from section **B.**
- Any additional question(s) answered will **not** be marked.
- **All** necessary working **must** be shown clearly.
- Begin each answer on a sheet of paper.
- Squared paper is provided.
- Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.
- In numerical work, take acceleration due to gravity, $g = 9.8\ ms^{-2}$.

**SECTION *A*: (40 MARKS)**

*Answer* **all** *the questions in this section.*

1. Kadijat noted the weights, $x$ grams of 30 chocolate buns. Her results are summarized by $\sum(x-k) = 315$ , $\sum(x-k)^2 = 4022$ ,where $k$ is a constant. The mean weight of the buns is 50.5 grams.

(i) Find the value of $k$. ***(03 marks)***

(ii) Find the standard deviation of $x$. ***(02 marks)***

2. The acceleration of a particle is $-10\mathbf{j}$. If the particle starts at (0,80) and moving with a velocity of $15\mathbf{i}$,

a) Find the velocity at time t, t=5s. ***(03 marks)***

b) Given that at time $T$, the particle is at $(x,0)$, calculate the values of $x$ and $T$.

***(02 marks)***

3. Use trapezium rule with six strips to estimate; $\int_0^{\pi} \sqrt{(1+\sin x)}\, dx$.

Truncate your answer correct to **three** significant figures. ***(05 marks)***

4. A car of mass 1000kg working at a constant rate of 160kW is moving with a constant speed of 20m/s up a plane inclined at an angle of $30^0$ to the horizontal. Find the magnitude of the resistance to the motion. ***(05 marks)***

5. The resistance of a wire at different temperature is as follows:

| | | |
|---|---|---|
| Resistance(Ω) | 24 | 42 |
| Temperature (°C) | 15 | 51 |

Use linear interpolation or extrapolation to estimate the:

(i) Temperature corresponding to 35 Ω. ***(03 marks)***

(ii) Resistance whose value is equal to that of the temperature. ***(02 marks)***

6. A box $P$ contains 3 red and 5 black balls, while another box $Q$ contains 6 reds and 4 black balls. A box is chosen at random and from it a ball is picked and put into another box. A ball is then randomly drawn from the later. Find the probability that;

(i) Both balls are red. ***(03 marks)***

(ii) First ball drawn is black. ***(02 marks)***

7. Four forces $\begin{pmatrix}2\\1\end{pmatrix}N$, $\begin{pmatrix}-1\\3\end{pmatrix}N$, $\begin{pmatrix}4\\-2\end{pmatrix}N$ and $\begin{pmatrix}-5\\-2\end{pmatrix}N$ acts on a particle at (1,1), (2,0), (2,3) and (−1,1) respectively. Show that the forces reduce to a couple. ***(05 marks)***

8. The masses of meat cans are normally distributed with a standard deviation of 18g. A random sample of 25 cans had a mean mass of 456g.

Find the 97.5% confidence interval for the mean mass of all the meat cans.

***(05 marks)***

**SECTION B: (60 MARKS)**

*Answers any* **five** *questions from this section.*

**All** *questions carry equal marks.*

9. The numbers of male and female candidates admitted at a certain university in a certain year to offer different courses *A,B,C,D,E,F,G,H,I* and *J* were as follows:

| Course | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ | $I$ | $J$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Male | 66 | 54 | 60 | 70 | 62 | 46 | 74 | 58 | 80 | 58 |
| Female | 50 | 38 | 54 | 68 | 60 | 32 | 62 | 46 | 70 | 49 |



a) Calculate the average number of
i) Males,
ii) Females,

Admitted to these courses that year.
*(04 marks)*

b) Represent the given data on a scatter diagram. *(02 marks)*
c) Determine the rank correlation coefficient for the above data and comment on your result at 0.05 level of significance. *(06 marks)*

10. a) If the period of a simple harmonic motion is 8s, and the particle oscillates through a distance of 1.2m on each side of the central position. Find;

i) The maximum velocity,
ii) The velocity when the particle is 0.6m from the central position. *(06 marks)*
b) A particle is moving with SHM in a straight line and takes 3s to perform a complete oscillation. Its furthest distance from the Centre is 1.2m.

i) Determine its maximum acceleration,

ii) Determine the new amplitude if the particle receives a blow when at its furthest point that drives it with the initial velocity of $0.6\pi\ ms^{-1}$. *(06 marks)*

11. a) The numbers $x = 1.5$, $y = -2.85$ and $z = 10.345$ were all rounded off to the given number of decimal places as indicated. Find the range within which the exact value of $\frac{1}{x} - \frac{1}{y} + \frac{y}{xz}$ lies. *(07 marks)*

b) Two decimal numbers $X$ and $Y$ were approximated with errors $E_1$ and $E_2$ respectively. Show that the maximum possible relative error in the approximation of the product $X^2Y$ is $2\left|\frac{E_1}{X}\right| + \left|\frac{E_2}{Y}\right|$ *(05 marks)*

12. a) The weights of a group of males are normally distributed with mean 80kg and variance 6.76kg$^2$. If a random sample of 16 of these men is selected, find the probability that the mean is less than 78.5kg. ***(04 marks)***

b) A football match may be either won or lost by the home team on assumption that no draw is made. The home team is twice as likely to win as to lose the match. If 72 games are played, find the probability that the home team will win;

i) Exactly 50 games,

ii) not more than 40 games. ***(08 marks)***

13. a) (i) Show that the equation $3^{2x} - 49 = 0$ has a real root between 1 and 2. ***(02 marks)***

ii) Show that the newton Raphson's formula for approximating the root of the equation is:

$$x_{n+1} = \frac{1}{\ln 9}\left[x_n \ln 9 + 49(3^{-2x_n}) - 1\right]$$ ***(03 marks)***

b) Draw a flow chart that;

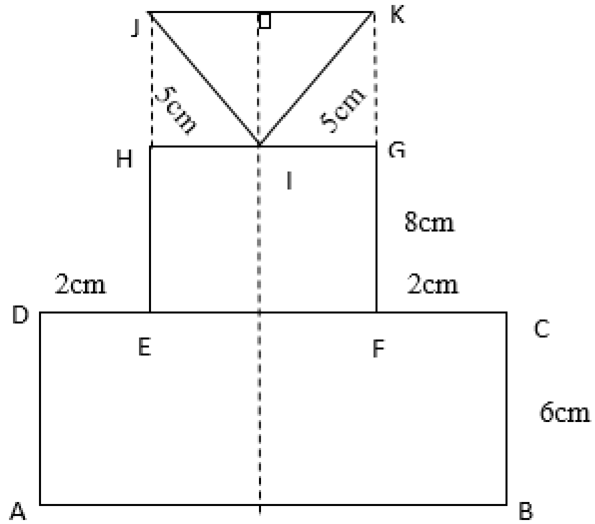
i) reads the initial approximation $x_0$ of the root,

ii) Computes and prints the root correct to **three** decimal places. ***(04 marks)***

d) Taking $x_0 = 1.75$, perform a dry run to find the root of the equation. ***(03 marks)***

14. a) Three particles of weights 2 $W$, 3 $W$, and 5 $W$ are located at the points with position vectors $\binom{1}{1}$, $\binom{2}{-3}$, *and* $\binom{4}{1}$ respectively. Find the coordinates of their Centre of gravity. ***(04 marks)***

b) A composite lamina *ABCDEFGHIJK* is made of a rectangular lamina *ABCD* 12cm by 6cm, a square lamina *EFGH* of side 8cm and triangular lamina *IJK* welded to the square lamina at I as shown below.




i) Find the distance from $AB$ to the position of Centre of gravity of the composite lamina. ***(06 marks)***

ii) If the lamina is suspended from $B$, find the angle $AB$ makes with the vertical. ***(02 marks)***

15. A game consists of tossing 4 unbiased coins simultaneously. The total score is calculated by giving 3 points for each head and 1 point for each tail. The random variable $X$ represents the total score.

a) Show that the probability of $P(X = 8) = \frac{3}{8}$ ***(04 marks)***

b) Copy and complete the table given below for the symmetrical probability distribution of $X$. ***(04 marks)***

| $x$ | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|
| $P(X = x)$ | | | $\frac{3}{8}$ | | |

c) Calculate the variance of $X$. ***(04 marks)***

16. Ship $A$ initially at a point with position vector $\binom{4}{2}$ is moving with a speed of $12kmh^{-1}$ in the direction $30^0$ East of North, while ship $B$ initially at a point with position vector $\binom{6}{10}$ is moving with a speed of $4kmh^{-1}$ due East. Find;

a) The velocity of $A$ relative to $B$. ***(04 marks)***

b) The shortest distance between the two ships in the subsequent motion and the time for which it occurs. ***(08 marks)***

**END**

# APPLIED MATHEMATICS SET EIGHT (P425/2)

**INSTRUCTION TO CANDIDATES**

- *Answer **all the eight** questions in section **A** and any **five** from section **B**.*
- *All working **must** be shown clearly.*
- *Begin each question on a fresh sheet of paper.*
- *Graph paper is provided*
- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

**SECTION A (40 MARKS)**

1. In a certain inter university tournament; 35% watched football but not cricket, 10% watched cricket but not football and 40% did not watch either game. Find the probability that they watched football, given that they did not watch cricket. (5marks)

2. A particle performs vertical simple harmonic motion of period 4 seconds and amplitude 5m. The ends of the path are points A and A' with A above A', and O is the centre of the motion. If the particle starts from rest at A when t = 0, determine its position when;

(i) t = 0.5 seconds (03 marks)

(ii) t = 1.3 seconds (02 marks)

3. Use the trapezium rule with 6 ordinates to estimate $\int_0^1 e^{x^2}\, dx$
Correct to two places of decimal. (05 marks)

4. The probability that a seed chosen at random form the bag will germinate is $\frac{4}{7}$. If 150 seeds are chosen at random from the bag, calculate the probability that less than 90 seeds will germinate. (05 marks)

5. A ship P is moving due west at 12kmh$^{-1}$. The velocity of a second ship Q relative to P is 15kmh$^{-1}$ in a direction 30$^0$ west of south. Find the velocity of ship Q.(05 marks)

6. A fraction $y = f(x)$is tabulated for various values of $x$ as shown below;

| $x$ | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 |
|---|---|---|---|---|---|
| $y$ | 3.70 | 3.82 | 4.15 | 4.51 | 5.07 |

Use linear interpolation to estimate the value of
(i) $y$ at $x = 1.15$ (03 marks)
(ii) $x$ for which $y = 4.40$ (02 marks)

7. Forces of magnitude 4N and 3N act along the sides AB and AD respectively of a square ABCD of side 2m. IfO is the midpoint of DC, calculate the perpendicular distance of the line of action of their resultant from O. (05 marks)

8. The table below shows the mock examination marks and the A level grades obtained by students in a certain year;

| Marks in mocks | 76 | 41 | 78 | 59 | 14 | 29 | 61 | 86 | 32 | 64 | 51 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Grades in A level | A | B | B | C | D | E | B | A | D | C | E |

Calculate the rank correlation coefficient of the performance of students. Comment on your results. (05 marks)

## SECTION B (60 MARKS)



9. In a certain college farm, maize is sold in bags of mean weight 40kg and standard deviation 2kg. If the weights of the bags were normally distributed, find the;

(i) Probability that the weight of any bag taken at random will lie between 41.0 and 42.5kg. (04 marks)

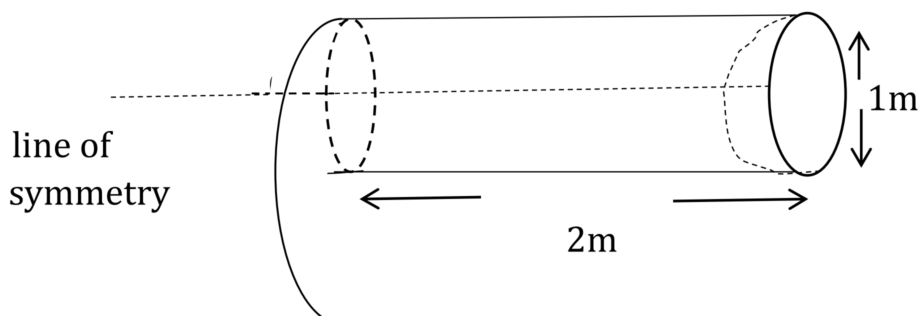
(ii) Percentage of bags whose weight exceeds 43kg. (04 marks)

(iii) Number of bags rejected out of 500 bags purchased by a retailer whose consumers cannot accept any bag whose weight is below 38.5kg. (04 marks)

10. (a) Show graphically that the equation $e^{2x} + 4x - 5 = 0$ had only one real root between 0 and 1. (04 marks)

(b) Use the Newton – Raphson iterative method to find the root of the equation in (a) above giving your answer correct to 2 decimal places. (08 marks)

11. The diagram below shows a uniform right circular solid cylinder with a radius 0.5m and length 2metres.




A solid hemisphere of the same radius is cut from one end of the cylinder, removed, and then attached to the un cut plane face of the cylinder forming a new solid. Find the centre of mass of the new solid. (12 marks)

12. A continuous random variable X has p.d.f given by

$$f(x) = \begin{cases} Kx^2(1-x) & ; \quad 0 \le x \le 1 \\ 0 & ; \quad otherwise \end{cases}$$

Find the;

(i) Value of the constant $K$. (03 marks)

(ii) Expectation of X. (03 marks)

(iii) mode (03 marks)
(iv) $P(0.4 < x < 0.6)$ (03 marks)



13. (a) Show that the iterative formula for finding the fourth root of a number N is given by

$$x_{n+1} = 3\left\{\frac{Xn}{4} + \frac{N}{12x_n^3}\right\}, \text{n} = 0, 1, 2, \text{----}(03 \text{ marks})$$

(b) Draw a flow chart that reads in $x_0$ and N, computes and prints the fourth root and N after three iterations and gives the root correct to 2 decimal places. (05 marks)

(c) Perform a dry run for N = 99.1, $x_0 = 3$. (04 marks)

14. A tennis player hits a ball at a point 0, which is at a height of 2m above the ground and at a horizontal distance 4m from the net, the initial speed being in a direction of $45^0$ above the horizontal. If the ball just clears the net which is 1m high,
(a) Show that the equation of path of the ball is $16y = 16x - 5x^2$. (04 marks)
(b) Calculate the;
(i) Distance from the net at which the ball strikes the ground. (04 marks)
(ii) Magnitude and direction of the velocity with which the ball strikes the ground. (04 marks)(Use g = $10ms^{-2}$)

15. The following cumulative frequency table refers to the ages of members of an under 30's' social club;

| Age (yrs) | Cumulative frequency |
|---|---|
| Under 10 | 0 |
| 14 | 5 |
| 16 | 15 |
| 17 | 26 |
| 18 | 48 |
| 19 | 70 |

(a) Draw a cumulative frequency curve and use it to estimate the
(i) median age
(ii) Semi- interquartile range. (06 marks)

(b) Calculate the
(i) mean age (03 marks)
(ii) Standard deviation of the club members. (03 marks)

16. A particle P of mass 8kg rests on a smooth horizontal table and is attached by a light inelastic strings to particles Q and R of mass 2kg and 6kg respectively. The strings pass over light smooth pulleys on opposite edges of the table so that Q and R hang freely. If the system is released from rest,

(a) Determine the
(i) acceleration of the particles
(ii) Tensions in the strings. (06 marks)

(b) After falling a distance of 1m from rest, particle R strikes an inelastic floor and is brought to rest. Determine the further distance that Q ascends before momentarily coming to rest. (06 marks)
(Assume that the length of the strings are such that P remains on the table and Q does not reach it ).

END

# APPLIED MATHEMATICS SET NINE (P425/2)

**INSTRUCTIONS TO CANDIDATES:**

- Answer all the **eight** questions in Section A and **five** from section B.
- All necessary working must be shown clearly.
- In numerical work take g to be $9.8ms^{-2}$.
- Mathematical tables with a list of formulae and squared papers may be used.
- Extra numbers will not be marked.

## SECTION A (40 MARKS)

1. A and B are independent events in a sample such that P $(A^{|}) = 0.6$ and $P(A\cup B) = 0.8$. Find:
   (i) P (B) (3 marks)
   (ii) $P(A\cup B^1)$ (2 marks)

2. A constant horizontal force of 85N causes a particle of mass 10kg to move across a rough horizontal plane, a distance of 5m from a speed of $5ms^{-1}$ to $10ms^{-1}$. Find the;
   (a) Acceleration of the particle (2 marks)
   (b) Coefficient of friction between the particle and the plane.(3 marks)

3. A car consumed fuel amounting to shs. 14,800, Shs. 15,600, Shs. 16,400 and Shs. 17,200 in covering distances of 10km, 20km, 30km and 40km respectively Estimate the;
(a) Cost of fuel consumed for a distance of 45km (3 marks)
(b) Distance travelled if fuel of shs. 16,000 is used (2 marks)

4. A continuous random variable X is uniformly distributed over the interval $a \le x \le \beta$. Given that $E(X) = 2$ and $P(X \le 3) = \frac{5}{8}$. Find the;
   (a) Values of $a$ and β (4 marks)
   (b) *p.d.f* of $X$ (1 mark)

5. One end of a light inextensible string of length 75cm is fixed to a point on a vertical pole. A particle of mass 1.2kg is attached to the other end of the string. The particle is held in equilibrium 21cm away from the pole by a horizontal force, $P$ Newtons. Find the;
   (i) Tension in the string (3 marks)
   (ii) Magnitude of $P$ (2 marks)

6. Real numbers $A$ and $B$ are rounded off to give numbers $a$ and $b$ with maximum possible errors of $e_A$ and $e_B$. Show that the maximum possible relative error made in computing $AB$ is $\left|\frac{e_A}{a}\right| + \left|\frac{e_B}{b}\right|$. State any assumptions made. (5 marks)

7. The data below shows the ages $X$ of patients and number of days taken, $Y$ to recover from a particular disease.

| $X$ | 55 | 51 | 62 | 66 | 72 | 59 | 78 | 55 | 62 | 70 |
|---|---|---|---|---|---|---|---|---|---|---|
| $Y$ | 34 | 44 | 49 | 49 | 48 | 43 | 51 | 41 | 46 | 51 |

(a) Calculate the rank correlation coefficient for the data (4 marks)

(b) Comment on the significance of the age on the number of days taken by the patient to recover fully at 1% level of significance. (1 marks)

8. A truck of mass 4m kg moving with a velocity of 54kmh$^{-1}$ makes a head on collision with a car of mass m kg moving with velocity of 36kmh$^{-1}$. If the truck moves in the same direction with the car embedded in it after collision, find the;

(a) Common velocity after collision (2 marks)

(b) Loss in kinetic energy (3 marks)

## SECTION B (60 MARKS)

**Attempt only five questions**

9. The time, $x$ seconds spend by each of a random sample of 100 customers at an Automated Teller Machine (ATM) are summarized in the table below.

| Time (seconds) | Frequency density |
|---|---|
| $10 \leq x \leq 15$ | 0.4 |
| $15 \leq x \leq 25$ | 0.8 |
| $25 \leq x \leq 30$ | 3.4 |
| $30 \leq x \leq 35$ | 5.2 |
| $35 \leq x \leq 45$ | 2.4 |
| $45 \leq x \leq 50$ | 3.2 |
| $50 \leq x \leq 60$ | 0.6 |
| $60 \leq x \leq 80$ | 0.05 |

(a) Draw a histogram and use it to estimate the mode. (4 marks)

(b) Calculate the;

(i) Mean time,
(ii) Semi-interquartile range of the time spent at the ATM (8 marks)



10. A force $24t\mathbf{i} - 12\mathbf{j}$ Newtons acts on a particle of mass 2kg initially at rest at point (-4,3). Find the;
(a) Position vector of the particle after t seconds. (7 marks)
(b) Work done by the force in the time interval $t = 1$ to $t = 2$ seconds. (5 marks)

11. (a) Use the trapezium rule with six ordinates of estimates $\int_2^1 \sin^2 x\,dx$, correct to three significant figures. (6 marks)
(b) Determine the error made in your calculation in (a) above and suggest how this error can be reduced. (6 marks)

12. The marks of 500 candidates is Subsidiary Mathematics are normally distributed with a mean of 45 marks and standard deviation of 20 marks Find the;
(a) Percentage of candidates who scored at least 68 marks. (4 marks)
(b) Number of candidates who will pass if the pass mark is 35. (4 marks)
(c) The lowest mark for a point if 290 candidates score a point in Subsidiary Mathematics. (4 marks)

13. (a) A body of mass 5kg is in limiting equilibrium on its own when it is placed on a rough incline plane. If the angle is friction is $\tan^{-1}\left(\frac{3}{4}\right)$. Find the minimum force acting parallel to the incline that will just move the body up the incline. (6 marks)
(b) *ABCD* is a square of side 6*cm*. Forces of magnitudes *10N, 12N, 15N, 7N* and $5\sqrt{2}N$ act along *AB, BC, CD, DA* and *DB* respectively in the direction shown by the order of the letters. Show that the forces reduce to a couple. (6 marks)

14. (a)(i) Show that the equation $x^3 = 5x + 1$ has a root between $x = 2$ and $x = 2.5$
(ii) Use linear interpolation to estimate the root to two decimal places. (5 marks)

(b) Given the two iterative formulae;

$$x_{n+1} = \frac{2x_n^3+1}{3x_n^2-5} \text{ and } x_{n+1} = \sqrt{5 + \frac{1}{x_n}}$$

Use the root in (a) (ii) above to deduce with a reason the more suitable formula. Hence give the root, correct to 2 d.ps. (7 marks)

15. A discrete random variable $Y$ has a $p.d.f$ given as;

$$f(y) = \begin{cases} ky & ;y = 1,2 \\ k(6 - y) & ;y = 3,4 \\ k & ;y = 5,6 \end{cases}$$

where $k$ is a constant. Determine the value of;

(a) Value of $k$ hence f (y) (4 marks)

(b) $E(Y)$ (3 marks)

(c) $P(Y) < \left(\frac{5}{Y}\right) \geq 3$ (5 marks)

16. (a) if a stone falls past a window 2.45m high in 0.5 seconds, find the height from which the stone fell. (4 marks)

(b) A body $P$ is projected vertically upwards with velocity 28ms$^{-1}$. Two seconds later another body $Q$ is projected vertically upwards from the same level with velocity 21ms$^{-1}$. Find the velocity of each body when they are at the same height. (8 marks)

**END**

# APPLIED MATHEMATICS SET TEN (P425/2)

**INSTRUCTIONS TO CANDIDATES:**

- Answer **all** questions in **section A** and any **five** from **section B.**
- All necessary working must be shown clearly.
- Silent non – programmable scientific calculators and mathematical tables may be used.

**SECTION A (40 MARKS)**

1. $A$ *and* $B$ are events such that $P(A^1 U B) = \frac{2}{5}$; $P(A \cap B) = \frac{3}{10}$.

Find $P(B/A)$. ***(05 marks)***

2. A particle, accelerating uniformly, moves with an average velocity of 8ms$^{-1}$ for 4 seconds. If its final velocity is 12ms$^{-1}$. Calculate the;

(i) distance covered

(ii) Acceleration of the particle. ***(05 marks)***

3. Give that $x = 4.8\ (1dps)$; $y = 3.25\ (2dps)$ find the interval within which the exact value of $x - y$ lies. ***(05 marks)***

4. Five people are picked at random from a group of 7 men and 3 women. Calculate the probability that at least 2 are men. ***(05 marks)***

5. A particle is projected from a point O on a horizontal ground with a speed of 20ms-1 at an elevation $tan^{-1}\frac{4}{3}$. Calculate the;

(i) time of flight,

(ii) height risen in a $\frac{1}{3}$ $of\ the\ time\ of\ flight$. ***(05 marks)***

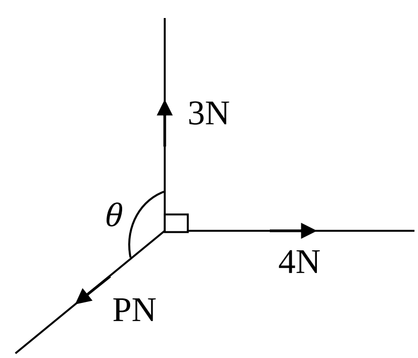
6.

| Subject | Ranks | | | | | | |
|---|---|---|---|---|---|---|---|
| Economics | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Maths | 2 | 4 | 3 | 5.5 | 1 | 5.5 | 7 |

Use the table to calculate the Spearman's rank correlation for the performance of the two subjects. Comment on your answer. ***(05 marks)***

7. Use the Neaton – Raphson method to compute $\sqrt{3}$ to 4 dps. ***(05 marks)***

8.




The 3 forces are in equilibrium, find the values of **P** and **Q**. ***(05 marks)***

**SECTION B (60 MARKS)**

9. (a) Given that $X$ $and$ $Y$ are approximate values with respective errors $\Delta X$ $and$ $\Delta Y$, show that the maximum error in $\frac{X}{Y}$ $is$ $\left|\frac{X}{Y}\right|\left\{\left|\frac{\Delta X}{X}\right| + \left|\frac{\Delta Y}{Y}\right|\right\}$ ***(07marks)***

(b) Given $X = 4.8$, $Y = 3.56$ corrected to the given number of decimal places; using the results in (a) above, or otherwise compute the maximum error in $\frac{X - Y}{X + Y}$. ***(05 marks)***

10. The table shows the distribution of heights of pupils in a school.

| Height (cm) | $0- < 50$ | $50- < 90$ | $90- < 100$ | $100- < 120$ | $120- < 160$ |
|---|---|---|---|---|---|
| Frequency | 8 | 16 | 20 | 32 | 4 |

(a) Construct a histogram for this data, and use it to find the mode.
***(07 marks)***

(b) Calculate the number of pupils whose heights lie between 80cm and 116cm.
***(05 marks)***

11. A boat is travelling north wards at $80km^{-1}$ when a wind starts to blow east wards at $60kmh^{-1}$. ***(05 marks)***

(a) Find the resultant velocity of the boat.

(b) Calculate the direction in which the boat must be steered so as to remain on its original course, and compute the resultant speed of the boat in this case.
***(07 marks)***

12. In a large group of patients 75% suffer from malaria.

(a) Ten patients are picked at random from the group, find the probability that between 4 and 9 are malaria patients. ***(05 marks)***

(b) Forty eight patients are picked at random, calculate the probability that;

(i) exactly 4,

(ii) At most 26 are malaria patients. ***(07 marks)***

13. A body of mass 2.5kg is placed on a rough inclined plane of angle $tan^{-1}\ \frac{4}{3}$. calculate the;

(a) Minimum force parallel to the plane that will keep the body at rest, the coefficient of friction being 0.5. ***(07 marks)***

(b) The acceleration of the body if it is released to move down the plane.
***(05 marks)***

14. (a) Show that the positive root of the equation $x^3 - 2x - 1 = 0$ lies between 1 and 2; use linear interpolation to find the first approximation of the root. ***(04 marks)***

(b) Construct a flow chart based on the Newton – Raphson algorithm for computing the root of the equation in (a) above. Perform a dry run of your flow chart. ***(08 marks)***

15. A boat 100 km North East of a ferry, is travelling North wards at $60kmh^{-1}$. At that instant, the ferry is travelling at $45\sqrt{2}\ kmh^{-1}$ due North West. Calculate the;

(a) Velocity of the ferry relative to the boat. ***(06 marks)***

(b) Shortest distance between the vessels. ***(06 marks)***

16. The marks obtained by 2000 UNEB candidates in Maths Paper 2 of a certain year were normally distributed with a mean of 64. The records showed that 60% of the candidates scored above 50.

(a) Calculate the standard deviation of the candidates' marks. ***(04 marks)***

(b) Find the pass mark, if 75% of the candidates passed the paper. ***(04 marks)***

(c) Calculate the number of candidates that scored between 45 and 55 marks. ***(04 marks)***

**END**

# APPLIED MATHEMATICS SET ELEVEN (P425/2)

**INSTRUCTIONS TO CANDIDATES**

- *Answer **ALL** the eight questions in Section A and any **FIVE** from Section B.*
- *All necessary working must be shown clearly*
- *Mathematical tables with a list of formulae and squared papers are provided*
- *In numerical work, take g to be $9.8ms^{-2}$*
- *Include the allocation table on your answer sheet*
- *Draw double margins on each of the answer sheets / page to be used.*

## SECTION A (40MARKS)



1. Two events $A$ and $B$ are such that $P(A) = 0.7$and $P(\bar{A} \cap \bar{B}) = P(\bar{A} \cup \bar{B}) = 0.2$.
   Determine the; i) $P(B)$ ii) $P(\bar{A}/B)$ (5marks)

2. A particle of mass 2kg is acted upon by a force of magnitude 21N in the direction $2i + j + 2k$. Find in vector form the;

i) Force

ii) Acceleration hence its magnitude. (5marks)

3. A certain student from S.6 of a certain school recorded the following set of points.

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | -5.5 | -3.0 | 1.2 | 3.4 | 6.0 |

Use linear interpolation or extrapolation to estimate;

i) $y$ when $x = -0.76$
ii) $x$ when $y = 7.8$ (5marks)

4. A discrete random variable X has the probability distribution function given by;

| $x$ | 5 | 8 | 9 | 11 | 12 |
|---|---|---|---|---|---|
| $p(X = x)$ | $\boldsymbol{a}$ | 0.1 | $\boldsymbol{a}$ | 0.4 | 0.1 |

Where $\boldsymbol{a}$ is a constant. Find the;

i) Value of $a$
ii) $E(5x - 7)$ (5marks)

5. Car A is travelling at $35ms^{-1}$ along a straight horizontal road and accelerates uniformly at $0.4ms^{-2}$. At the same time, another car B moving at $44ms^{-1}$and accelerating uniformly at $0.5ms^{-2}$, B is 200m behind A, find the time taken before B overtakes Car A. (5marks)

6. Use the trapezium rule with five ordinates to estimate $f(x) = x + tanx$ from $x = 1$ to $x = 1.4$ correct the value to 3 decimal places. (5marks)

7. The table below shows the time recorded in minutes when Aeroplanes pass through a point of observation at a certain city.

| Time | 50- | 60- | 70- | 80- | 90- | 100- | 110-120 |
|---|---|---|---|---|---|---|---|
| Frequency | 5 | 3 | 8 | 7 | 10 | 8 | 9 |

Calculate the;

i) median

ii) Number of Aeroplanes whose time exceed the median value. (5marks)

8. Forces of magnitude 20N, 12N and 30N act on a particle in the directions due South, east and N$40^0$E respectively. if the fourth force holds the particle in equilibrium; Determine the;

i) Magnitude

ii) Direction of the forth force. (5marks)

## SECTION B (60MARKS)

*Answer any **five** questions in this Section.*

9. The probability density function of a random variable X is given by;

$$f(x) = \begin{cases} \frac{4}{5}x \ ; & 0 < x < \\ \frac{2}{3}(3-x); & 1 < x < \\ 0 & otherwise \end{cases}$$

a) Sketch the function f(x) and show that the area =1.

b) Find the mean of x.

c) Determine the cumulative distribution function ($F(x)$). (12marks)

9. a) Two ships A and B are observed from a coast guard station and have the following displacements velocities and times.

| Ship | Displacement | Velocity | Time(t) |
|---|---|---|---|
| A | $(i + 3j)km$ | $(i + 2j)kmhr^{-1}$ | 12:00hours |
| B | $(i + 2j)km$ | $(5i + 6j)kmhr^{-1}$ | 13:00hours |

Find the time when the two are closest to each other.

b) If at 13:00hours ship A changed its velocity to $\left(\frac{11}{3}i + 2j\right)kmhr^{-1}$, show that they collide and find the time and position of collision. (12marks)

11. a) show graphically that the equation $f(x) = 1nx - sinx - 2$ has a root between $x = 3$ and $x = 4$ andestimate the initial approximation $(x_0)$ to 1 decimal place.

b) Using the $x_0$ above and the Newton Raphson method find the root correct it to 3 decimal places. (12marks)

12. The table below shows the speeds $(y)$ in seconds and the number of errors $(x)$ in the typed scripts of 12 secretaries of a certain institution.

| Secretaries | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Errors$(x)$ | 12 | 24 | 20 | 10 | 32 | 30 | 28 | 15 | 18 | 40 | 27 | 35 |
| Speed$(y)$ | 130 | 136 | 120 | 120 | 153 | 160 | 155 | 142 | 145 | 172 | 140 | 157 |

a) Construct a scatter diagram, draw the line of best fit and comment hence estimate x when $y = 142$.

b) Giving rank 1 to the fastest secretary and the secretary with the fewest errors calculate the rank correlation co-efficient and comment at 5% level of significance. (12marks)

13. A uniform lamina is in form of a square ABCD of side 2cm. E is a point on $AD$ such that $ED = xcm$, if protion EDC is removed, find the expressions of the location of centre of gravity from AB and from AD, taking AB as the positive y-axis and AD as the positive x-axis. (12marks)

14. a) Given that the numbers x=4.2, y=16.02 and Z=25 are rounded off with corresponding percentage errors 0.5, 0.45 and 0.02 calculate the errors of x, y and Z.

b) Hence find the maximum value, the minimum value, absolute error, relative error and percentage error in $\frac{xy}{z}$. (12marks)

15. The speed of cars passing a certain point on a motorway can be taken to be normally distributed. Observations show that of cars passing the point, 95% are travelling at less than $85kmhr^{-1}$ and 10% are travelling at less than $55kmh^{-1}$. Determine the;

a) Mean and standard deviation of the distribution.

b) Proportion of cars that travel at more than $70kmhr^{-1}$ and the percentage it takes. (12marks)

16. A light inextensible string has one end attached to aceiling, the string passes under a smooth moveable pulley of mass 2kg and then over a smooth fixed pulley, the particle of mass 5kg is attached at the free end of the string, the sections of the strings not in contact with the pulleys are vertical, if the system is released from rest and moves in a vertical plane, determine the;

i) Accelerations of the 2kg and 5kg masses

ii) Tensions of the 2kg and 5kg masses.

iii) Distance moved by the system in 1.5 seconds. (12marks)

**END**

# APPLIED MATHEMATICS SET TWELVE (P425/2)

**INSTRUCTIONS TO CANDIDATES:**

- Answer **all** the **eight** questions in section **A** and any **five** from section **B.**
- Any additional question (s) answered will not be marked
- All necessary working **must** be shown clearly
- Begin each answer on a fresh sheet of paper
- Graph paper is provided
- Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.
- In numerical work, take g to be $9.8\ ms^{-2}$.

**SECTION A (40 MARKS)**

1. Events A and B are independent such that $P(A \cap B) = ¼$ and $P(A \cup B) = {}^{3}/_{4}$.

Find the $P(A)$ and $P(B)$. *(05 marks)*

2. Given that w = 28.114, x = 7.136, y = 41.8446 and z = 3.6827, each number being rounded off to the given number of decimal places, find the percentage error in $\frac{w}{x} - \frac{y}{z}$ correct to 2 significant figures. *(05 marks)*

3. A parcel of weight 10N lies on a rough plane inclined at an angle of $30^0$ to the horizontal. A horizontal force of magnitude P Newtons acts on the parcel. If the parcel is in equilibrium and on the point of slipping up the plane and the normal reaction of the plane on the parcel is 18N.Find the

*(a)* value of P *(03 marks)*

(b) Co – efficient of friction between the parcel and the plane. *(02 marks)*

4. The continuous random variable X has a probability density function given by;

$$f(x) = \begin{cases} \frac{3}{4}(1 + x^2) & , \quad 0 \leq x \leq 1 \\ 0 & , \quad otherwise \end{cases}$$

Find $P\left(x > \frac{1}{2} \,/\, x > \frac{3}{4}\right)$ *(05 marks)*

5. A particles of mass 2kg is projected from a point at the bottom of a rough plane inclined at $\tan^{-1}\frac{4}{3}$, to the horizontal. If the coefficient of friction between the particle and the planes is $\frac{4}{7}$ and the particle first comes to rest at point A, calculate the distance OA. *(05 marks)*

6. Use the trapezium rule with 5 strips to evaluate

$\int_0^4 e^{\sqrt{x}}\, dx$, correct to 3 decimal places. *(05 marks)*

7. The masses, to the nearest kilogram, of 200 students were recorded as in the table below.

| Mass (kg) | Frequency |
|---|---|
| 41 – 50 | 21 |
| 51 – 55 | 62 |
| 56 – 65 | 55 |
| 66 – 70 | 50 |
| 71 – 85 | 12 |

Draw a histogram to represent this information and use to estimate the model mass.

*(05 marks)*

8. A ship P is moving due west at $12kmh^{-1}$. The velocity of a second ship Q relative to P is $15kmh^{-1}$ in a direction $30^0$ west of South. Find the velocity of ship Q. *(05 marks)*

## SECTION B (60 MARKS)

9. The table below shows the distribution of random sample of marks of a group of candidate during an examination.

| Marks | Frequency |
|---|---|
| $0- < 10$ | 10 |
| $10- < 20$ | 25 |
| $20- < 40$ | 30 |
| $40- < 60$ | 42 |
| $60- < 70$ | 16 |
| $70- < 95$ | 15 |

(a) Calculate the

*(i)* Mean mark *(03 marks)*

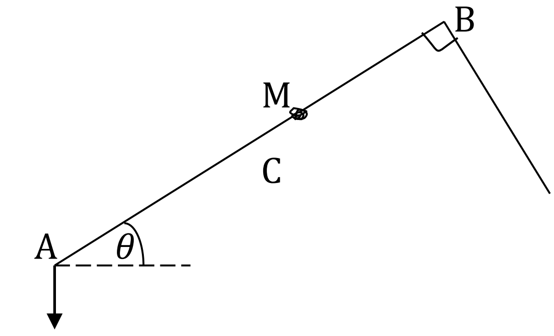
*(ii)* Standard deviation of the distribution. *(03 marks)*

(b) If the sample was taken from a population which is approximately normally distributed, determine the 99.5% confidence limits for the population mean mark, correct to two decimal places. *(06 marks)*

10. (a) Show that the root of the equation $f(x) = In\ x - sinx - 2 = 0$ lies between 3 and 4. *(03 marks)*

(b) By using the Newton – Raphson method, find the root of the equation in (a) above correct to 2 decimal places. *(03 marks)*

11. The diagram below shows a uniform rod AB of weight 200N and length 5m which is smoothly hinged at its midpoint to a fixed pivot M. A particle of weight 400N is attached to the rod at A.




The other end B has a light string attached which is fastened to a fixed point C. If the rod is in equilibrium with AB making an angle 0 with the horizontal where $\cos\theta = \frac{3}{4}$ and the angle ABC is $90^0$. Calculate the;

(i) Tension in the string. *(05 marks)*

(ii) magnitude of the resultant force exerted by the pivot on the rod *(07 marks)*

12. The table below summarises the results of the distance run by each members of an athletics club in training for a particular week.

| Distance to member (km) | Number if athletics |
|---|---|
| 31 – 40 | 10 |
| 41 – 45 | 15 |
| 46 – 50 | 20 |
| 51 – 55 | 70 |
| 56 – 57 | 64 |
| 58 – 60 | 24 |
| 61 – 70 | 20 |
| 71 - 90 | 10 |

Calculate the;

(a) mean time *(03 marks)*

(b) variance *(03 marks)*

(c) mode *(03 marks)*

(d) median *(03 marks)*

13. The information below gives a system of tax (T) calculation for the amount of money (A) earned annually by employees of a certain international company.

| **Annual earnings (A)** | **Tax (T)** |
|---|---|
| < £2000 | zero |
| ≥ £2000 *but* < £5000 | 2% of A |
| £5000 ≤ A | £60 plus 5% of the amount over £5000 |

*(a)* Draw a flow chart using the above data, given that the algorithm stops when 200 count (as) are made. *(10 marks)*

*(b)* Calculate the tax for an employee who earns £6000 annually. *(2marks)*

14. (a) A particle projected from a point O on a horizontal ground moves freely under gravity and hits the ground again at A. Taking O as the origin, the equation of the path of the particle is $240y = 80x\sqrt{3} - x^2$, where x and y are measured in metres . Calculate the;

(i) Initial speed and angle of projection.

(ii) distance OA

(take $g = 10\text{ms}^{-2}$) *(06 marks)*

(b) A ball is kicked with a velocity of $10\text{ms}^{-1}$ at an angle of $40^0$ to the horizontal towards a wall which is 7m away.

*(i)* Find how far up the wall the ball hits. *(04 marks)*

*(ii)* Calculate the speed of the ball when it hit the wall. *(02 marks)*

*(iii)* Determine the direction the ball is moving when it hits the wall. *(02 marks)*

15. An experiment consists of removing 2 sweets one at a time without replacement from a box containing 3 red and 4 blue sweets.

(a) If A is the event that both sweets picked are of the same colour, find the probability that event A occurred. *(02 marks)*

(b) If the experiment is repeated 20 times, find the probability that event A occurred

(i) Between 20 and 35 times. *(03 marks)*

(ii) at least 25 times *(04 marks)*

16. A system consists of a fixed pulley B and a movable pulley A . A light, in extensible string passes over pulley B and curves pulley A on one end and a particle of mass 6kg on the other. A second, similar string passes over pulley A and carries particles of mass 2kg and 4kg. If the pulleys are light and smooth, find the;

(a) Tensions in the strings. *(07 marks)*

(b) Accelerations of the three masses. *(05 marks)*

## SEMINAR QUESTIONS

## PART A: PURE MATHEMATICS (P425/1)

1. (a) (i) Given that $a^3+b^3 = 6ab(a+b)$ , prove that $ln\left(\frac{a+b}{3}\right) = \frac{1}{2}(lna + lnb)$

(ii) Find x in the equation $5^{\log_{25} x} = 3^{\log_{27} 2x}$

(iii) Show that $\log(x+y) = \log x + \frac{1}{3}\log\left(1+\frac{3y}{x}+\frac{3y^2}{x^2}+\frac{y^3}{x^3}\right)$

(iv) Find the root of the equation $2 + \log\sqrt{1+x} + 3log\sqrt{1-x} = log\sqrt{1-x^2}$

(b) (i) Given that the roots of the equation $px^2 + qx + q = 0$ are $(\propto_1 - p)$ and $(\propto_2 - q)$. If $(\propto_1 - p) - (\propto_2 - q) = 1$

Show that $\frac{\propto_1}{\propto_2} = \frac{2p^2+(p-q)}{2pq-(q+p)}$

(ii) Use raw echelon reduction to solve the following equations simultaneously:

2x = 5 + y - z

-3y = 2 – x – 2z

4z = -3 – 2x - y

(c) (i) Given that $f(x) = (x-\propto)^2 g(x)$, show that $f^1(x)$ is divisible by $(x-\propto)$

(ii) A polynomial $p(x) = x^3 + 4ax^2 + bx + 3$ is divisible by $(x-1)^2$. Use the result in (c) (i) above to find the values of **a** and **b**, hence solve the equation $p(x) = 0$

2. (a) Prove by induction $4^{n+3} - 3n - 10$ is divisible by 3 for all positive integral values of n.

(b) **NASACA** opened up a bank account with shs. 50,000, she deposits the same amount every year and makes no withdrawals. After how many years will she accumulate more than one million shillings on her account if the bank offers 5% compound interest per annum?

3. (a) **MASSAPPE** is a common word used by Ugandans today

(i) How many possible arrangements of the letters in the word **MASSAPPE** without restriction.

(ii) How many possible arrangements of the letters in which the two **A's** are together.

(iii) How many possible arrangements of the letters in which the two **A's** are separated.

(b) A teacher in **MASS** is to form a team of competitors in mathematics. How many teams of 6 competitors can be formed from a group of 7 boys and 5 girls , if :

(i) Each team should have atleast 3 boys and a girl.

(ii) Each team contains at most 3 girls.

4. (a) Find the coefficient of $x^3y^4$ in the expansion of $(2x - 3y)^7$

(b) Prove that if x is so small that its cube and higher powers can be neglected, $\sqrt{\frac{1+x}{1-x}} \simeq 1 + x + \frac{x^2}{2}$, by taking $x = \frac{1}{9}$, show that $\sqrt{5} \simeq \frac{181}{81}$

5. (a) If $x$ is real and $y = \frac{5x^2+8x+4}{x^2+x}$, show that the curve y cannot lie between - 4 and +4

(b) Solve the following inequalities

(i) $\left|\frac{x^2-4}{x}\right| \leq 3$

(ii) $|x + 3a| > 2|x - 2a|$

(iii) $\frac{x+2}{x-3} < \frac{x+5}{x-5}$

(c) Sketch the curve $y = \frac{3x+3}{x(3-x)}$ by clearly finding the turning points and asymptotes

6. (a) (i) Show that $\frac{16!}{9!7!}+\frac{2\times16!}{11!6!}+\frac{16!}{11!5!}=\frac{18!}{11!7!}$

(ii) The ratio of the twenty third term of an A.P to the third term exceeds the ratio of the twenty second term to the fourth by 0.5. Given that the sum of the first 25terms is 225, find the first term and common difference of the two progressions which satisfy these conditions.

(b) (i) The first term of a G.P is 3 and the ratio of the third term to the seventh term is 3:4 find the ninth term

(ii) Given a geometric series sin2x+-sin2xcos2x+sin2xcos²2x+……………………

Find the common ratio and prove that the sum to infinity is $tanx$

8. (a) (i) Use De- moivre's theorem to show that:

$15\sin^5\theta = sin5\theta - 5sin3\theta + 10sin\theta$

(ii) Prove that $3i+2$ is a root to the equation. $Z^4-5Z^3+18Z^2-17z+13=0$, and hence find all other roots of this equation.

(b) (i) Given that $Z_1=6\left(cos\frac{5}{12}+isin\frac{5}{12}\pi\right)$, find $Z_1Z_2$ and $\frac{Z_1}{Z_2}$ in the form $x+yi$

(ii) Calculate the principle argument of $\frac{\left(1+i\sqrt{3}\right)^5}{(1-i)^3}$

(iii) Express $Z=\frac{7+4i}{3-2i}$ in the form $p+qi$ where p and q are real.

9. (a) If $Z_1=2+5i$, $Z_2=1-3i$ and $Z_3=4-i$. Determine in both Cartesian and polar forms the value of $\frac{Z_1+Z_2}{Z_1+Z_2}+Z_3$ **correct to 3dps**

(b) (i) Describe the locus given by $|Z+2i|=|2Zi-1|$

(ii) Evaluate $(1+i)^8$

(iii) If $Z=x+iy$ and $|Z-4|\le 3$, determine the least and greatest value of Z

(c) Given that $z=z+iy$ and $arg\left(\frac{z}{z-6}\right)=\frac{\pi}{2}$. Show that the locus of Z is

$x^2+y^2-6x=0$

INTEGRATION

**1. (a) Find the following integrals**

(i) $\int (lnx)^2 dx$

(ii) $\int \frac{2dx}{\sqrt{1-x^2}\cos^{-1}(x)}$

(iii) $\int e^{(e^x+x)}\, dx$

(iv) $\int 2^{2^{2^x}}.2^{2^x}.2^x\, dx$

(v) $\int \frac{d\theta}{\sqrt{1-sin\theta}}$

(vi) $\int xtan^2xdx$

(vii) $\int \frac{2cosx+9sinx}{3cosx+sinx}dx$

**(b) Evaluate the following:**

(i) $\int_0^{\frac{\pi}{2}} \frac{cos^4x}{1+sinx}$

(ii) $\int_0^{\frac{\pi}{12}} tan^3 3xdx$

(iii) $\int_0^{\frac{\pi}{2}} sin7xcos5xdx$

(iv) $\int_0^1 \frac{3-x}{(x+1)(x^2+1)}$

(v) $\int_0^1 \tan^{-1}(2x)dx$

**(c) Show that:**

(i) $\int x\sin^{-1}(x)\,dx = \frac{1}{4}(2x^2-1)\sin^{-1}(x) + \frac{1}{4}x\sqrt{(1-x^2)} + c$

(ii) $\int_{\sqrt{3}}^{\infty} \frac{dx}{x\sqrt{1+x^2}} = -\frac{1}{2}ln3$

(iii) $\int \frac{x}{2x^2-x+1}dx = \frac{1}{4}\ln(2x^2-x+1) + \frac{1}{6}\tan^{-1}\left(\frac{4x-1}{3}\right) + c$

(iv) $\int_1^4 \frac{\log_e x}{\sqrt{x}}dx = 8\log_e 2 - 4$

2. (a) Form and state the order of the formed de given the equations below

(i) $\frac{y}{Ax^2+Bx} = 1$

(ii) $x = \frac{1}{\cot(Ay)}$

(b) Solve the following differential equations

(i) $\frac{dy}{dx} = \cos(x - y)$ given that, $y(\pi) = 0$

(ii) $\frac{dy}{dx} = e^{10t+12y}$when y = 0 , x =1

(iii) $\frac{dy}{dx} + 2xtanx = sinx, y\left(\frac{\pi}{4}\right) = 0$

(iv) $x^2\frac{dy}{dx} = x^2 + y^2 + xy$ given that y= 0 when x = $\frac{\pi}{4}$

(v) $\frac{dy}{dx} = 4x - 3y + 2xy - 6$

(c) The population of criminals in Nansana grows at a rate given by the equation $\frac{1}{x}\frac{dx}{dt} = (b - ax)$ given that originally there was one criminal in Nansana. Show that;

$$\left(\frac{x}{b-ax}\right)^{1/b} = \left(\frac{1}{b-a}\right)^{1/b} e^t$$

3. (a) Milk tea poured in metallic cup loses heat due to a steady breeze at a rate which is proportional to its temperature $\theta$ and also gains heat from a hot fire source directed to it at a rate proportional to time, t,

(i) Write down the differential equations for the temperature $\theta$

(ii) Show that at any time t, $\theta = At + B + Ce^{-kt}$

(b) Find the mean value of $y = \frac{\tan^{-1}(x)}{1-x^2}$ for $0 \le x \le \frac{\pi}{4}$

(c) Determine the volume of the solid generated when the area of the segment cut off by $y = 6$ from the curve $y = x^2 + 2$ is rotated about $y = 6$

4. (a) Differentiate the following from 1st principles

(i) $y = e^{kt}$

(ii) $y = xlnx$

(iii) $x^4+sinx^2$

(iv) $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

(b) (i) Given that $y = (\sin^{-1} x)^2$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$

(ii) If $y = 1 - cos\theta$ and $x = sin\theta$, prove that $\left(\frac{d^2y}{dx^2}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^2$

5. (a) Differentiate the following w.r.x

(i) $\frac{3^x+6}{9^x \log_e 3}$

(ii) $\frac{e^{lncosx} sinx}{\sqrt{cotx}}$

(b) Given that $x = sec\theta + tan\theta, y = cosec\theta + cot\theta$. Show that $x + \frac{1}{x} = 2sec\theta$ and $y + \frac{1}{y} = 2cosec\theta$. Hence show that $\frac{dy}{dx} = -\left(\frac{1-y^2}{1+x^2}\right)$

(c) If y= $(secx + tanx)^2$, show that $cosx\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 2tanx$

6. (a) A water tank of a uniform cross sectional area 2cm$^2$ has a tap at the back. When the tap is opened water flows out at a rate proportional to the depth of water in the tank.

(i) Show that $\frac{dh}{dt} = -\lambda h$

(ii) If the depth of water is 1cm when the tap is opened, find the time it will take until the depth is 50cm, assume $\lambda = \frac{1}{50}$

(b) The curve has the equation $x - y = (x + y)^2$. It is also given that the curve has only one turning point.

(i) Show that $1 + \frac{dy}{dx} = \frac{2}{2x+2y+1}$

(ii) Hence or other show that $\frac{d^2y}{dx^2} = \left(1 + \frac{dy}{dx}\right)^3$

(iii) Deduce whether this turning point is maximum or minimum

(c) Find an approximate value for $\sqrt[3]{64.96}$

(d) Use Maclaurins theorem to expand $(1 - 3x + 5x^2)$ up to the third non – zero term

1. **(a) Solve:**

(i) $4sin^2\theta + 8cot^2\theta - 5cosec^2\theta = 0$ for $0 \leq x \leq 360^0$

(ii) $cot^2\theta - 2cot\theta cosec\theta = 0, 0 \leq x \leq 360^0$

(iii) $2^{tan^2x+8} - 32(2^{tanx}) + 1 = 0$ where $0 \leq x \leq 180^0$

**(b) Prove that :**

(i) $\left(\frac{1+sin2x}{1-sin2x}\right)^{1/2} = \frac{1+tanx}{1-tanx}$

(ii) In any triangle ABC, $tan\frac{A}{2}tan\frac{B}{2} + tan\frac{B}{2}tan\frac{C}{2} + tan\frac{C}{2}tan\frac{A}{2} = 1$

(iii) $\frac{sin2x-1-cos2x}{2(1-sin2x)} = \frac{1}{tanx-1}$

2. (a) (i) Show that $\sin^{-1}\left(\frac{3}{5}\right) - 2\tan^{-1}\left(\frac{1}{5}\right) = \sec^{-1}\left(\frac{65}{63}\right)$

(ii) Prove that $\tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}$

(iii) Show that $\sin[2\sin^{-1}(x) + \cos^{-1}(x)] = \sqrt{(1-x^2)}$

(b) Express $2\sqrt{3}sin\theta cos\theta + 2\cos\theta^2$ in the form $a\sin(2\theta + a) + b$, hence solve the equation $2\sqrt{3}\,sin\theta cos\theta + 2cos^2\theta = 3$ for $0^0 \leq \theta \leq 360^0$

(c) Find a positive value of $\theta$ that satisifies the equation $\tan^{-1} 3\theta + \tan^{-1}\theta = \frac{\pi}{4}$

1. (a) Find the Cartesian equation of the plane

$$r = (1 + 3\lambda + 3\mu)i + (1 + \lambda + 4\mu)j + (\mu + \lambda)k$$

(b) (i) Find the equation of a line passing through points A (1, 2, 5) B(2,1,0) and C(5,3,2)

(ii) Determine the perpendicular distance from the points B(1,13) to the line:

$$\frac{x+4}{2} = \frac{y+1}{3} = \frac{z-1}{3}$$

(c) (i) Find the perpendicular distance between the planes $6x - 3y + 2Z + 4 = 0$ and $6x - 3y + 2z - 12 = 0$

(ii) Find the acute angle between the planes in (c) (i) above.

2. (a) Find the equation of the plane passing through the origin and parallel to the lines

$$\frac{x+2}{3} = \frac{y-1}{4} = \frac{z+1}{5} \text{ and } \frac{x-3}{4} = \frac{y-2}{-5} = \frac{z+1}{1}$$

(b) Find the possible values of **t** given that the vectors $ti + 4j + (2t + 1)k$ and $(t + 2)i + (1 - t)j - k$ are perpendicular to each other.

(c) Show that the line $\frac{x-2}{2} = \frac{y-2}{4} = \frac{z-3}{3}$ and plane $r\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} = 4$ are parallel and fin the perpendicular distance of the line from the plane.

(d) The position vectors of vertices of triangle are O, r and S where O is the origin , show that its area (A) is given by A = $\frac{\sqrt{|r|^2|s|^2-(r.s)^2}}{2}$, Hence find the area of a triangle where r = $\binom{2}{3}$ and s = $\binom{1}{-4}$

1. (a) Show that the Cartesian equation whose polar equation is given by $r^2 = a\,(sec2\theta\ + 1)$ is $x^4 - y^4 = 2ax^2$

(b) Find the angle between the lines:
$ax - by + c = 0$ and $(a - b)x + (a + b)y + d = 0$

(c) Given that the curve $y^2 = x^3$

(i) Obtain the equation of the normal at the point $(t^2, t^3)$

(ii) Show that the equation of the normal at the point where t = ½ is
$32\,x + 24y - 11 = 0$

(iii) Find the perpendicular distance for the point (-1, 2) to this normal

2. The point $p(ap^2, 2ap)$ lies on a parabola $y^2 = 4ax$ the normal at p cuts the x- axis at Q

(a) Find the coordinates of Q

(b) R divides PQ externally in the ratio 2 : 1 ,
Show that the locus of $R$ is $y^2 + 16a^2 = 4ax$

3. (a)The points $P(ap_1{}^2, 2ap_1)$ and $Q(ap_2{}^2, 2ap_2)$ are on a parabola $y^2 = 4ax$ . OP is perpendicular to **OQ** where O is the origin , show that $p_1p_2 + 4 = 0$

(b) The normal to the rectangular $xy = 8$ at a point (4,2) meets the asymptotes at M and N. Find the length **MN**.

# PART B: APPLIED MATHEMATICS (P425/2)



1. (i) Use the trapezium rule to estimate the area of $y = 3^x$ between the x – axis , x =1 and x = 2 using 5 strips. **Give your answer to 4 s.f**

(ii) Find the exact value of $\int_1^2 3^x dx$

(iii) Find the percentage error in the calculations (i) and (ii) above

2. (a) In an experiment to measure the rate of cooling of an object, the following temperature ($^0$C) against times, T(s) were recorded.

| **Temperature** | 80 | 70.2 | 65.8 | 61.9 | 54.2 |
|---|---|---|---|---|---|
| **Time, T** | 0 | 10 | 15 | 20 | 30 |

Use linear interpolation / extrapolation to find:

(i) The values of $\theta$ when T = 18s

(ii) T when $\theta = 60^0$

(b) In the table below is an extract of part of $logx$ to base 10, $\log_{10} x$

| $\boldsymbol{x}$ | 80.0 | 80.20 | 80.50 | 80.80 |
|---|---|---|---|---|
| $\boldsymbol{\log_{10} x}$ | 1.9031 | 1.9042 | 1.9059 | 1.9074 |

(i) Use linear interpolation / extrapolation to estimate $\log_{10} 80.759$

(ii) The number whose logarithm is 1.90388

3. (a) If $z = sinx$ Determine the expressions for the absolute error and maximum relative error

(b) Given that the error in measuring an angle is $0.5^0$. Find the maximum possible error in $\frac{sinx}{cosx}$ if $x = 30^0$

4. (a) It is known that an examination paper is marked in such a way that the standard deviation of the marks is 15.1. In certain school 80 candidates take the exam and they have an average mark of 57.7 find,

(i) 95% and

(ii) 9% confidence limits for the mean mark in the examination

(b) The table below shows the distribution of weights of a random sample of the 26 times taken from large consignment.

| **Weight** | 97 | 98 | 99 | 100 | 101 | 102 |
|---|---|---|---|---|---|---|
| **Frequency** | 2 | 1 | 2 | 3 | 6 | 2 |

Assuming the weights are normally distributed determine the 93% confidence interval for the mean weight of all the tins.

5. A continuous random variable x has the distribution function

$$f(x) = \begin{cases} 3kt\left(1 - \frac{x^2}{3}\right) & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

Determine :

(i) the value of k

(ii) the probability density function of x

(iii) the mean of x

6. (a) For the probability that a female and male teachers pass the intervals $^1/_3$ and $^2/_5$ respectively. Assuming these events are independent, determine the probability that both pass the intervals.

(b) In a certain university , 75% of the students are full – time students , 45% of the students are female , 40% of the students are male full time students ., find the probability that;

(c) A student chosen at random from the students in the university is a part time student.

(d) A student chosen at random from all students in the university is female and part time student.

(e) A student chosen at random from all the female students in the university is a part time student.

7. The table below shows the marks obtained by students of maths in a certain school

| Marks | Number of students |
|---|---|
| 30 - < 40 | 02 |
| 40 - < 50 | 15 |
| 50 - < 55 | 10 |
| 55 - < 60 | 11 |
| 60 - < 70 | 30 |
| 70 - < 80 | 29 |
| 90 - < 100 | 3 |

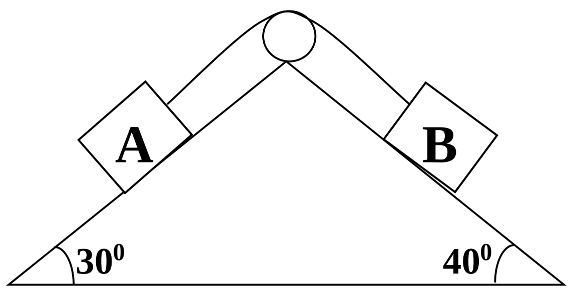
(a) Calculate e the mean and standard deviation

(b) Draw an O - give for the data

(c) From the graph determine

(i) The median
(ii) The 90th percentage
(iii) Interquartile range

8.




Two particles A and B rest on an inclined plane of a fixed triangular wedge as shown above. A and B are connected by a light inextensible string which passes over a smooth fixed pulley at C. The faces of the wedge are smooth and A and B are both 7kg masses.

Find the force exerted by the string on the pulley at when the system is moving freely with both particles in contact with the wedge.

9. A smooth inclined plane of length **L**, and height **h**, is fixed on a horizontal plane. Show that the velocity with which a particle must be projected down the plane from the top in order that it may reach the horizontal plane in the same time as a particle let fall from the top is

$$u = \frac{L^2 - h^2\left(\sqrt{\frac{g}{2h}}\right)^{1/2}}{L}$$

10. (a) A particle moving with a velocity $(2i + 3j)ms^{-1}$ is accelerated uniformly at the rate of $3ti - 2j)ms^{-1}$ from the origin. Find the:
(i) Speed reached by the particle at t = 45
(ii) Distance travelled by the particle at t = 25
(b) A particle moving in straight line with uniform acceleration, a, passes a certain point with a velocity , u, three seconds later another particle moving in the same line with constant acceleration $\frac{4}{3}a$, passes the same point with a velocity $\frac{1}{3}u$. The first particle is over taken by the 3 second when their velocities are respectively 8.1 and 9.3ms$^{-1}$. Find the values of $\boldsymbol{u}$ and $\boldsymbol{a}$ and also the distance travelled from the point.

# PRINCIPAL MATHEMATICS SEMINAR QUESTIONS

## PURE MATHS P425/1

### (A)Coordinate Geometry (02 questions)

1. (a) Find the equation of the line that passes through the point of intersection of the lines 3x-2y=4 and 2x+2y-6=0 and makes angle $135^0$ with the horizontal.

(b) find the locus of the point p(x,y) which moves such its distance from a point A(1,2) is twice its distance from the line 2x-y=4.

2. The normal to the parabola $y^2=4ax$ at a point $P(at^2,2at)$ meets the axis of the parabola at S.If SP is produced beyond P to R such that SP=PR,Show that the equation of locus of R is $y^2 =16a(x+2a)$.

3. Find the length of the tangent to the circle $x^2+y^2+4x-3y=0$ from a point (1,2).Hence find the point where this tangent meets the circle.

4. Given parametric equations $x = 2 + 3\cos\theta$, $y= 1 + 4\sin\theta$,

(i) Show that the equation represents an ellipse.

(ii) Find the centre and length of the major axis.(iii) Find the equation of the normal.

4. Find the equation of the tangents to the hyperbola $x = 4t$ and $y = 4/t$ which pass through the point (4,3).

### (B)Trigonometry (02 questions)

1. If $\sin A=\frac{12}{13}$ and $\cot B=\frac{3}{4}$, where A is acute and B is reflex, find the value of 2SecA-$Cosec^2$B.

2. Solve for θ, $2\cos\theta\cos2\theta - \cos\theta + 1 = 0$ where $0 \leq \theta \leq \pi$. hence or otherwise solve the equation $t^4 + 8t^3 + 2t^2 - 8t + 1 = 0$

3. In triangle *ABC,* prove that; $\frac{bc}{ab+ac} = \frac{Cosec(B+C)}{CosecB+Cosec\ C}$

4. Show that $\tan\frac{\pi}{8} = -1 \pm \sqrt{2}$

(b) Solve the equation $\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = 8$.

5. Prove that $\sin^4\theta+\cos^4\theta=\frac{1}{4}(\cos4\theta+3)$

## (C)Vectors (02 questions)



1. Find the equation of the plane containing the lines $r_1$=(3-2λ)**i** -5λ**j**+(-2+3λ)**k** and $r_2$=-4**i**-(4+μ)**j**+(3+2μ)**k** passing through the origin.

2. Points O,P and Q are vertices of a triangle OPQ where O is the origin and vectors OP=**p** and OQ=**q**.R lies on OP produced such that OR=3OP,S is the midpoint of OQ and T divides the line PQ in the ratio 1:3.S is joined to T and to R.

(a) express in terms of vectors p and q: (i)PT (ii) ST (iii) SR

(b)If nST=kTR, find the values of n and k

3. Given that r and s are inclined at $60^o$, t is perpendicular to r + s and | r | = 8,| s | = 5, | t | = 10, find i) | r + s + t | ii) | r – s |

4. Find the distance of the point C (10, -3, -2) from the line which satisfies the coordinates A (4,-1, 2) and B (-2, 2,-4)

5. Given AB $=\begin{pmatrix}1\\2\\3\end{pmatrix}$ and AC $=\begin{pmatrix}4\\-1\\2\end{pmatrix}$ are vectors. Find a vector which is perpendicular to both AB and AC.

## (D)Algebra (04 questions)

1. Show clearly on the argand diagram the region represented by locus of the complex number such that $\frac{\Pi}{2} \leq \text{Arg}(\frac{z+2i}{z-i}) \leq \frac{\Pi}{2}$ .

3. Find the maclaurin's series for ln (1+x) as far as the term in $x^4$ .Deduce the maclaurin's series for ln(1-x) and hence for ln$(\frac{1+x}{1-x})$.Using the first four non-zero terms of this latter series find an approximation of ln2 .

3. (a)Coach Simon has 18 players and 3 goal keepers to choose from a football team of Buddo s.s to play in copa-cola in Jinja.In how many ways can he select his team if(i)all players are fit(ii)one sticker must be on the team and one goal keeper has injury problems.

(b)The URA has to produce number plates consisting of 4 letters of the alphabet and 3 digits .given that letter U must start and any letter can end a given number plate, how many number plates can be manufured by URA?

4.(a)Evaluate $\sum_{n=1}^{11}(1.2)^n + 1.2n$

(b)Solve the simultaneous equations.

$y\log_2 8 = x$

$2^x + 8^y = 8192$

5.(a) Given that the first three terms in the expansion in ascending powers of $x$ of $(1-8x)^{1/4}$ are the same as the first three terms in the expansion of $\left(\frac{1+ax}{1+bx}\right)$, find the values of $a$ and $b$. Hence, find an approximation to $(0.6)^{\frac{1}{4}}$ in the form $p/q$.

(b) Evaluate the square root of 5-√3

6. Given the curve y=$\frac{x^2-7x+10}{x-6}$

(i) Show that for real x, y cannot be between 1 and 9

(ii) Hence determine the turning points.

(iii)State all the asymptotes and the intercepts

(iii) Hence sketch the curve

## (E) Analysis (06 questions)

1. $Find \int_1^2 tan^{-1}x\, dx$

2. A vessel holding flowers is of the shape formed by the revolution of the curve $5y^2=2x$ about the y-axis a complete turn .if its height is 2cm.find its volume.

3. Water runs at constant rate of $6cm^3s^{-1}$ in a vessel whose volume is obtained by rotating the area bounded by the curve $4y=x^2$ about the y-axis from y=0 to y=h cm

(i) Show that the volume of the vessel is $2\Pi h^2 cm^3$

(ii) Find the rate at which the water level is rising when the water has been running for 3s.

4. Psychologist believes that when a student is asked to recall a set of facts in an examination, the rate at which the facts are recalled is directly proportional to the number of relevant facts in his/her memory that have not yet been recalled .Given that $q_0$ is the total number of relevant facts in the student's memory and q is the total number of relevant facts that have been recalled after time t.

(i) Write down a differential equation describing this situation.

(ii) If $q_0$=1000, q=500 at t=0 and q=250 at t=2hrs. Solve your differential equation

(iii) Given that a student passes the examination if he or she recalls atleast 50% of the facts, show that at this rate of recall he or she cannot pass a 3 hours examination.

5. Solve the differential equation $xy\frac{dy}{dx}=y^2+x^2e^{\frac{y}{x}}$.

7. Given that $y = \ln(1+\sin x)$, prove that $\frac{d^2y}{dx^2}+e^{-y}=0$

8. (a)Differentiate cosec x from first principles.

(b)Evaluate $\int_0^{\pi/2}(3\sin^2 x + 2\cos^2 x)\,dx$

9. (a)Find (i) $\int \frac{1}{\sqrt{12+4x-x^2}}\,dx$ (ii) $\int \frac{2x^3}{8+x^8}\,dx$

(b)Evaluate $\int_1^3 \frac{x^2+1}{x^3+4x^2+3x}\,dx$

# PRINCIPAL MATHEMATICS SEMINAR QUESTIONS

## APPLIED MATHS P425/2

## (A)MECHANICS (06 questions)

### Kinematics (02 questions)

(general kinematics, linear and vertical motion, projectiles, relative motion and resultant velocity.)

1. A girl who swims at $5kmh^{-1}$ in still water wishes to cross a river 150m wide, flowing at 8kmh-1. By the aid of suitable vector diagrams, find the direction in which she should swim in order to reach the opposite bank;

(i) as soon as possible,

(ii) As little downstream as possible.

Hence determine how long she will take to cross, and how far she will be carried downstream in each case?

2. A particle of mass 5kg moves so that its position vector after t seconds is $r = (\cos 2t)i + (3+4\sin 2t)j$ m.(a) Find the Speed of the particle when $t=\pi/3$.

(b) Find ;(i)magnitude of the acceleration ,

(ii) Force acting on the particle when $t=\pi/2$

3. A stone is thrown from the top of a hill 100m high at angle of 30° below the horizontal and hits the ground 20m from the foot of the hill. Find,

(i)The initial speed of the stone.

(ii)Time taken to hit the ground.

4.A particle of unit mass moves along a smooth curve while accelerating at $(2t\mathbf{i}-\mathbf{j}+3t^2\mathbf{k})ms^{-2}$ .initially the particle is located at a point $(\mathbf{i}-3\mathbf{k})$m moving with a velocity $2\mathbf{j}ms^{-1}$.

(i) Find impulse after 2 seconds.

(ii) Velocity at any time t seconds.

(iii) Work done between t=1 to t=3.

3. (a)A ship A is travelling on a course of 060° at a speed of 30kmh-1 and a ship B is travelling at 20kmh-I.At noon B is 260km due east of A.

(i)Find the course B must take to come as close as possible to A.

(ii) Find the time when A and B are closest together and the shortest distance.

(b) Two motor cyclists M and N are travelling along straight roads" meeting at right angles to each other, with uniform speeds of30kmh-1and 40km-1 respectively towards point O, the cross-road . If M and N are initially 0.75km and 1.2km.from O, find, the shortest distance between the cyclist in the subsequent motion and the time taken for it to occur.

4. A cyclist was timed between successive trading centres P, Q and R, each 2 km a part. It took 5/3 minutes to travel from P to Q and 2.5 minutes from Q to R Find:

(i) The acceleration

(ii) The velocity with which the cyclist passes point P

(iii) How much further the cyclist will travel before coming to rest if the acceleration remains uniform.

5. Kampala is South of Gulu town, To a passenger in Gulu-Kampala bound bus traveling at 110km/hr the wind appears to be blowing in the direction 240·

When the bus reduces speed to 90kmlhr without changing direction;' the wind appear be blowing the direction 210.Find the true speed and the direction from which he wind is blowing.

## (b)Statics

(Ladder problems, hinged bodies and jointed rods, C.O.G, friction, resolving forces, couples and line of action).

1. The centre of a regular hexagon ABCDEF of side 2m is O. Forces of magnitude IN,2N,3N,4N,P and Q act along AB, BC, CD, DE, EF and FA respectively. Given that the resultant force of the six forces is of magnitude 3N acting in the direction parallel to EF;

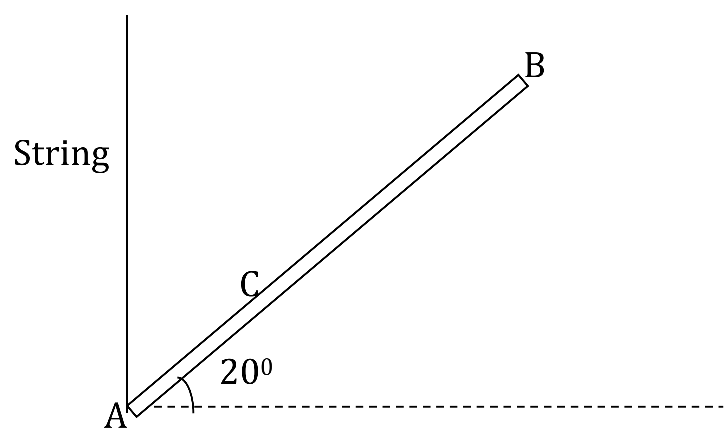
(i)Determine the values of P and Q.

(ii) Referring to OM as x-axis and OA as the y-axis where M is the midpoint of EF, find the equation of line of action of the resultant.

2. A constant force of 35N acting horizontally causes a particle of mass 2kg to move over a rough horizontal plane. The particle passes two points A and B, 5m apart with speeds of $5ms^{-1}$ and $l0ms^{-1}$respectively, find the magnitude of the frictional Resistance.

3. A particle is in equilibrium under the action of five forces of magnitude 6N, IIN, 8N and 15N acting in direction N 72°W, N 15°W, N 35°E and S 63°E respectively. Find the magnitude and direction of the fifth force.

4. The diagram below shows a uniform AB, of mass 0.8kg and length 10a supported at end A by a light inextensible vertical and rests in limiting equilibrium on a rough fixed peg at C where AC =7a.



Given that AB makes an angle of $20^0$ with the horizontal,

(a) Find the tension in the string. **Ans: T = 2.24N.**

(b) Calculate the magnitude of the normal reaction exerted by the peg on the rod at C.

**Ans: R = 5.9594N**

5. ABC is a uniform triangular lamina right angled at B. AB = 2t and BC = 3t. Show that the center of gravity of ABC is at a distance t from AB. The mid points P and Q of CB and cA respectively are joined and the portion PQC is cut off. Find the distance from AB and BC of the centre of gravity of the lamina ABPQ. When this lamina is suspended freely from the vertex A, AB, is at angle θ to the vertical. Find tan θ.

## (c) Dynamics Questions

(Connected particles, work, energy, Momentum and power, elasticity and S.H.M, circular motion).

1. A particle of mass 2kg is released from rest at a point A on the outer surface of a smooth fixed sphere of centre 0 and radius 0.6m. Given that OA makes an angle β with the upward vertical, find an expression for the speed at which the particle is travelling when it leaves the surface of the sphere.

2. Particles of mass 2kg and lkg are placed on an equally rough slope of a double inclined plane whose angles of inclination are $60^0$ and $30^0$ respectively. They are connected by a light inextensible string passing over a smooth pulley at the common vertex of the planes. If the heavier particle is on the point of slipping downwards, show that the angle of friction is $33.4^0$.

3. A car of mass 1800 kg ascends an incline at a constant speed of l4 $ms^{-1}$.Given that the frictional resistance is 400N and the engine is working at a rate of 17.5KW. (i) Calculate the angle that the incline makes with the horizontal. (ii) Assuming that the frictional resistance and the rate of working remain constant, determine the acceleration of the car on a level road at an instant when the speed is 28 $ms^{-1}$

4. In each minute, a pump draws 2.4$m^3$ of water from a well 5m below the ground and issues it at ground level through a pipe of cross section area 50cm2.

(a) Find the;

(i) Speed with which the water leaves the pipe.**(Ans:8m/s)**

(ii) Rate at which the pump is working.**(Ans:3.42kW)**

(b) By taking the fact that this pump is only 75% efficient, find the rate at which it must work.**(Ans:4.32kW)**

5. A car of mass 800kg is towing a trailer of mass 150kg on a level road .the frictional resistance to each vehicle amounts to 7N per kg of mass.

(a) Calculate the tension in the tow bar when the vehicles are travelling at a constant speed.

(b)the car and trailer now climb a slope of inclination 1 in 20, and if the frictional resistances are the same as before and the power of the engine is 50kW, calculate the;

(i) Maximum speed up the slope.

(ii)Acceleration when the speed is 54$kmh^{-1}$ .

6. A box of mass 4.9kg is released from rest from the top of a rough inclined planed of inclination 60° to the horizontal. If the coefficient of friction between the box and the plane is 0.025, find the kinetic energy of the box 2 seconds after release.

7. A particle moving with S.H.M, performs 10 oscillations per minute and its speed when at a distance of 8m from the centre of oscillation is 3/5 of its maximum speed. Find the;

(i) Amplitude

(ii) Speed of the particle when it is 6m from the centre.

**(B)NUMERICAL METHODS (04 questions)**

1. (a)(i)Show that the iterative formula based on Newton Raphson for finding the $Cos^{-1}(N)$ is given by $X_{n+1} = X_n + CotX_n - NcosecX_n$ , n=0,1,2,3,……

(ii)By taking $x_0$=1.2, evaluate $Cos^{-1}(0.4)$ to 4decimal places (b)Given the two iterative formulae;

$x_{n+1} = \frac{x_n^3-1}{5}$ and $x_{n+1} = \sqrt{\left(5+\frac{1}{x_n}\right)}$

Using $x_o = 2$ , deduce a more suitable formula for solving the equation. Hence use the formula once to find the second approximation of the root, correct to three decimal places.

2. (a)Show that the iterative formula for solving the root of the equation$x^3 - 3x + 4 = 0$ is given as $x_{n+1} = \sqrt{3 - \frac{4}{x_n}}$ n=0,1,2,3, ...

(b)(i) Draw a flow chart to illustrate the use of algorithm for computing and printing the root of the equation to 3 d.ps or after 3 iterations have been performed.

(ii) Taking the initial approximation of the root as $-2.13$ , perform a dry run for your flow chart.

3.(a)Use the trapezium rule with 7 ordinates to estimate $\int_1^2 \tan x\, dx$ correct to 4 decimal places. Obtain the error in estimating this value and state two ways how this error can be reduced.

4 (a) Given that numbers A and B are approximated with errors $e_A$ and $e_B$ respectively.

State the maximum possible errors in (i)A-B (ii)AB

(b) If A=2.13±0.2 and B=0.95 computed with relative error 0.1, find the maximum possible error in (i) A-B (ii) AB

(c) The numbers x and y were rounded off to give X and Y with errors $e_1$ and $E_2$ respectively. Show that the maximum possible percentage error in $x\sqrt{y}$ is by given

$\left[\left|\frac{e_1}{X}\right|+\frac{1}{2}\left|\frac{E_2}{Y}\right|\right]$ x100.Hence or otherwise if x=2.53 and y=5.340, find the relative error in $x^2y^3$.

5. (a)Given the formula $x_{n+1} = \sqrt[3]{3 - x_n^{-2}}$ .Find the equation whose root is saught. Hence show that the equation has three real roots.

(b) Use linear interpolation to find the greatest root of the equation to 3 significant figures.

6. By taking the initial approximation of the root to 0.9, find the approximate value of $\log_5 4$ to 3 decimal places.

## (C)STATISTICS (06 questions)

1. The table below shows the indices and weights of making an item from components A,B and C in 2015 and 2018

| Components | Prince index | Weight |
|---|---|---|
| | | |
| A | 120 | 2 |
| B | 95 | 3 |
| C | 112 | 5 |

(a) Calculate the composite index for 2018.comment on your results.
(b) Obtain the cost of component A in 2018 if its cost in 2015 was shs. 65000/=

2. An experiment consists of removing 2 pens. One at a time with replacement from a box containing 3 red and 4 blue pens. If A is the event that both pens are of the same colour.
i. Find P(A)
ii. If the experiment is repeated 70 times, find the probability that event A occurred at least 25times.

3. Two tetrahedral dice both numbered 1 to 4 are thrown. If one dice is unbiased and the other is biased such that a four is twice as likely as any other number to show, find the probability that a sum of five is obtained.

4. The drying time of a newly manufactured paint is normally distributed with mean 110.5 minutes and standard deviation 12 minutes.

(a) Find the probability that the paint dries for less than 104 minutes.

(b) If a random sample of 20 tins of the paint was taken , find the probability that the mean drying time of the sample is more than 112 minutes.

(c) If a random sample of 5 tins is taken, find the probability that at least 4 tins will have a drying time of at least 115 minutes.
(d) A random sample of 16 tins taken from a different type of paint of standard deviation 15 minutes is found to have a mean time of 105.5 minutes, determine the 90% confidence limits the mean time of this type of paint .

5. The time taken to perform a particular task, t hours has probability density function given by

$$f(t) = \begin{cases} 10kt^2; & 0 \leq t \leq 0.6 \\ 9k(1-t); & 0.6 \leq t \leq 1.0 \\ 0; & \text{elsewhere} \end{cases}$$

where k is a constant

(a) Sketch f(t) and hence write down the most likely time

(b) Determine the;

(i)value of constant k

(ii) Expected time

(ii) Probability that the time will be between 24 and 48 minutes.

(c) Obtain $P(T \leq t)$ and sketch its graph.

6. The table below shows marks given to 6 students in sub-maths mock examination by 3 different examiners,

| Student | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Examiner 1 | 60 | 35 | 52 | 38 | 70 | 65 |
| Examiner 2 | 40 | 55 | 71 | 40 | 42 | 80 |
| Chief examiner | 55 | 60 | 41 | 63 | 73 | 76 |

(a) Calculate the rank correlation between Examiner 1and Examiner 2 with the chief examiner.

(b) State with a reason which of the examiners had a better correlation with the chief examiner.

7. Three events A, B and C are such that A and B are independent, A and C are mutually exclusive. Given that P(A) = 0.4, P(B) = 0.2, P(C)=0.3 and P(CnB) = 0.1 ,find

(i) $P(AUB^I)$ (ii) P(A/BUC).

8. The data below shows masses of objects in kg obtained.

| Mass(kg) | 20-24 | 25-29 | 30-34 | 35-39 | 40-49 | 50-54 | 55-64 |
|---|---|---|---|---|---|---|---|
| Frequency density | 0.4 | 1.2 | 1.4 | 2.2 | 1.8 | 1.6 | 1.2 |

Calculate the; (i) modal mass. (ii)mean mass. (iii) Number of objects whose mass is within central 70% range.

8. Box A contains 3 red and 4green apples, Box B contains 5red and 7 green apples. Two apples are picked at random such that when the first apple comes from box A, the second comes from box B and vice versa. Write down the probability distribution for the number of green apples picked and hence obtain the mean.

**THE END**